

Value-Based Multidisciplinary Optimization for Commercial Aircraft Design and Business Risk Assessment

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Traditional commercial aircraft design attempts to improve performance and reduce operating costs by minimizing takeoff weight. A better design approach also takes into account factors such as aircraft demand, market uncertainty, and development and manufacturing costs. This paper presents a design methodology that integrates an aircraft performance model with a program valuation technique based on real options theory to address uncertain market demand and managerial flexibility. The coupled performance/financial framework enables an integrated approach to technical design and programmatic decisions. In addition, the methodology provides a framework for specification of design requirements and for quantification of the financial implications associated with technical and business uncertainty. The methodology is demonstrated for an aircraft design example of the blended-wing-body concept. Comparing performance-optimized and value-optimized designs, we show that use of value as a design metric leads to a trade-off between aerodynamic efficiency and reduced manufacturing costs. Key findings demonstrate that traditional financial metrics cause the decision maker to focus overly on reducing costs in the short term. The stochastic methodology shows that a willingness to spend up-front money in the design process to ensure long-term profitability is a better strategy.

Nomenclature

C_L	=	lift coefficient
D	=	drag
ΔLC	=	adjustment in lifecycle cost
$E[NPV]$	=	expected net present value
F_t	=	objective function at time t
L	=	lift
M	=	Mach number
μ	=	rate of return
N	=	number of time periods
NPV	=	net present value
N_{seats}	=	number of seats
P	=	payload weight
Price	=	aircraft price
P_t	=	profit function at time t
Range	=	aircraft range
r_d	=	risk-adjusted discount rate
σ	=	demand volatility
s_t	=	state vector at time t
t	=	time
u_t	=	control vector at time t

Introduction

THE historical objective of minimizing gross take-off weight (GTOW) in aircraft design is intended to improve performance

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and subsequently lower operating costs, primarily through reduced fuel consumption. However, such an approach does not guarantee the profitability of a given aircraft design from the perspective of the airframe manufacturer. In an increasingly competitive market for commercial aircraft, manufacturers may wish to design for improved financial viability of an aircraft program in addition to technical merit before undertaking such a costly investment. The existing practice of designing aircraft from a technical perspective without simultaneously considering the impact on overall program value is suboptimal in a business sense.

To assess the long-term financial viability of an aircraft program, a value-based design approach is necessary. Such an approach should still account for performance while also incorporating tools to estimate the profitability of the program. Financial models, such as life-cycle cost analysis¹ and direct operating cost,² have been incorporated into multidisciplinary design optimization (MDO) of aircraft in the past by several researchers. Value metrics, such as net present value (NPV), internal rate of return (IRR), and return on investment (ROI), have also been considered as design objectives.³ The problem has also been approached as a multiobjective optimization balancing cost and performance.⁴

Traditionally, as in these cases, MDO has resulted in deterministic solutions for GTOW, value, or other objectives of interest. More recently, effort has been devoted to probabilistic multidisciplinary approaches to also address the issue of uncertainty. A key focus of this work is the idea of design risk, that is how technical or financial uncertainty affect performance and value. Uncertainty has been addressed in the form of design affordability and the balancing of technical performance, cost, and risk,⁵ and via probabilistic design to improve ROI⁶ and cost per passenger mile.⁷ Efforts have also been made to look at reducing risk throughout product development⁸ and to minimize risk using an approach based on estimation theory.⁹ For automotive applications, decision-based design has been used as a way to integrate technical and financial considerations using NPV as an objective function¹⁰ and using the technique of discrete choice analysis to construct a product-demand model.¹¹

Value metrics such as NPV are based on static valuations of the design. These metrics do not attempt to capture explicitly technical or financial uncertainties that may arise and, as such, do not properly account for the associated business risk of the program. Further, the

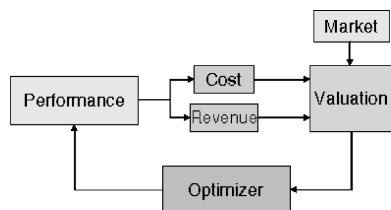


Fig. 1 Block diagram depiction of value-based MDO framework.

related issue of flexibility—that is, the ability of the manufacturer to make decisions in response to unexpected or changing conditions—is not considered. In the field of finance, considerable research has been performed to develop more sophisticated valuation techniques to address the shortcomings of traditional valuation techniques. Substantial literature exists describing real options theory, which provides a way to quantify the value of a product or strategy in the presence of uncertainty.^{12–14}

Drawing on improvements in valuation techniques, a stochastic dynamic programming (DP) framework has been developed for decision making in the context of commercial aircraft program design.^{15,16} The framework links aircraft performance, cost, and revenue models to provide an optimal program strategy and a quantitative valuation in terms of expected NPV of an aircraft program in an uncertain market. It has been shown that a stochastic valuation provides a comprehensive method to assess commercial aircraft programs.¹⁶ In that work, however, the valuation was applied a posteriori to aircraft designs resulting from a traditional performance-based optimization. Here, a new methodology is established using the stochastic valuation directly to make aircraft conceptual-design decisions via an optimization framework that combines technical design and value into a single problem. Applications of this value-based approach to design of the Boeing blended-wing-body (BWB) aircraft are presented herein. The business risk faced by the program due to potential sources of uncertainty can then be assessed for the resulting designs.

The framework illustrated in Fig. 1 demonstrates the approach to couple a performance model and associated optimization routine with empirically based financial models, a stochastic model of an uncertain market, and an algorithm for computing expected program value. A single program concept, incorporating technical design as well as financial parameters, can then be optimized in terms of specific performance or business goals, for example, minimizing GTOW or maximizing program value.

The next section describes the optimization framework, the performance and financial models, and the stochastic valuation methodology. A demonstration of the integrated methodology is then presented for a notional BWB program. In the following section, the specification of programmatic and design requirements is addressed. Examples for setting design range and speed of the BWB are presented. Application of the value-based methodology to assess business risk from technical and financial uncertainty is considered next. Finally, the paper concludes by summarizing key findings.

Optimization Framework

The design-optimization framework couples performance and financial models with an optimization routine as illustrated in Fig. 1. For design of the BWB aircraft, the WingMOD performance MDO framework is used.¹⁷ An initial design vector is provided to WingMOD to estimate aircraft sizing and performance characteristics. Its relevant outputs are then used by the cost and revenue models to approximate cost, price, and baseline demand figures for the design. The valuation module uses a stochastic DP algorithm, which accounts for market growth and uncertainty, to determine a set of optimal design decisions and the objective: expected NPV ($E[NPV]$). Optimization is carried out using a sequential quadratic programming algorithm to maximize $E[NPV]$.

Because of the extent of the BWB design problem, the performance-based optimization is carried out sequentially as a number of intermediate steps as described by Wakayama.¹⁷ These

steps progress the design to its overall solution in the following order (grouped into general categories entailing multiple steps): set the aircraft size and layout, set the performance (including range and speed), trim the aircraft and establish control limits, and balance the aircraft while minimizing GTOW. The value-based design optimization adds an additional step: maximize $E[NPV]$. Each suboptimization features its own objective, design vector with $O(100)$ variables, and $O(1000)$ constraints specific to the short-term goal. It is necessary to break up the optimization in this fashion for the optimizer to converge on a solution that meets all design criteria. Because of numerical limitations of the gradient-based optimization, an all-at-once optimization approach would make convergence to a feasible solution difficult (unless a very good initial design were provided). As discussed in detail by Peoples,¹⁸ the position in the sequence of the value-based optimization has some effects on the outcome. The best results were obtained by incorporating the maximum- $E[NPV]$ optimization step in the latter part of the sequence.

In the following sections, each of the components of the optimization framework are described in more detail.

Program Valuation

A number of metrics are available for assessing the value of the program, such as NPV, IRR, and ROI. There are advantages and disadvantages to each, as well as particular investment scenarios for which some metrics are better suited than others.

Deterministic Net Present Value

NPV is one of the most commonly used metrics in engineering program valuation. Deterministic NPV is computed by summing discounted future cash flows as follows¹⁹:

$$NPV = \sum_{t=0}^N \frac{P_t}{(1 + r_d)^t} \quad (1)$$

where t represents a future time period ranging from the current time ($t = 0$) to the final time period N and P_t is the profit function consisting of the difference between the revenues and costs of the undertaking in a given time period t . The risk-adjusted discount rate r_d accounts for both the opportunity cost of capital and the perceived risk inherent in a venture. Typically assumed values of r_d for an aircraft program may range from 12% to 20%.^{15,20} In general, a positive NPV indicates a sound investment, and a negative NPV means that a program should not be pursued. This approach is limited in some respects in its ability to provide a definitive valuation, however. NPV and other deterministic metrics are based on a static valuation of the design, because the expected cash flow must be assigned for each time period. The issue of flexibility—that is, the ability of the manufacturer to make decisions in response to changing conditions—is not considered. Further, uncertainty in future cash flows is handled only through choice of r_d . Finding or calculating a discount rate that includes the effects of risk is a difficult, and ultimately arbitrary, process. Using a high discount rate to reflect greater uncertainty does not properly account for risk and often leads to a pessimistic valuation, because cash flows in later years will contribute a small amount to the sum in Eq. (1).

Stochastic Valuation Methodology

A stochastic valuation approach based on real options theory addresses several of the shortcomings of a deterministic NPV calculation. Rather than relying on a risk-adjusted discount rate, demand uncertainty is addressed explicitly via a stochastic model based on empirically determined market volatility. The valuation is risk neutral—that is, it does not depend upon the risk preferences of the decision maker. To capture flexibility, the program is viewed as a series of investment decisions characterized by discrete program “modes,” including design, tooling, and manufacturing stages. The manufacturer may choose to pause or cancel the program when in certain modes, investing no more money if future market conditions appear unfavorable as determined by the demand model. A

successful aircraft program will by necessity be a dynamic venture, so accounting for the ability to make decisions as the program progresses is essential to assessing its profitability.

E[NPV] is calculated for this problem by solving Bellman's equation,

$$F_t(s_t) = \max_{u_t} \{P_t(s_t, u_t) + [1/(1 + r_f)]E_t[F_{t+1}(s_{t+1})]\} \quad (2)$$

where $F_t(s_t)$ is the value (objective function) at time t and state vector s_t , P_t is the profit in time period t as a function of the state vector s_t and the control vector u_t , and r_f is the risk-free discount rate (accounting now only for the time-value of money and not for risk). E_t is the expectation operator, providing in this case the expected value of F at time $t + 1$, given the state s_t and control u_t at time t . For the aircraft program modeled here, a time horizon of $T = 30$ years is considered. The state vector contains two elements: the quantity of aircraft demanded, which evolves stochastically, and the operating mode from the previous period. As discussed in detail by Markish,¹⁵ the operating mode describes the current status of the aircraft program (whether in design, tooling, low-capacity manufacturing, etc.). The control vector contains the decision to be made, in this case the program operating mode for the next period. The formulation of this stochastic valuation is described more fully in Refs. 15 and 16.

Equation (2) is then solved using a DP algorithm that starts at the final time period, $t = N$, and works backward to the current time, $t = 0$. The valuation module outputs program E[NPV], given by F_0 , and the corresponding set of optimal control decisions, $u_t(s_t)$. These control decisions represent the optimal decision strategy; that is, the decision rules specify the optimal value to which the control variable should be set, as a function of time and of all the state variables. In this case, given how long the program has been ongoing, the operating mode from past period, and the current market conditions (quantity demanded), the decision rule specifies what the operating mode should be for the next period. It is important to note that the expected NPV returned by the valuation module will never be less than zero, because zero E[NPV] can be obtained by choosing to always wait in the face of unfavorable design conditions. The DP solution will therefore not yield a strategy that, on average, produces a negative E[NPV]. This improved valuation technique provides a more appropriate measure of value than traditional financial metrics, because uncertainty is captured explicitly via the expectation operator and dynamic program flexibility is modeled through the control decisions.

Multidisciplinary Models

The DP problem described above requires annual cost and revenue estimates for the aircraft program. These values are derived from models that take the aircraft design parameters as inputs. The simulation model first uses the design vector to generate size and performance estimates for an aircraft concept. Then, relevant design values are used by the financial modules to calculate cost, price, and baseline demand for the resulting aircraft program as described next.

Performance Model

The WingMOD multidisciplinary performance model is used to calculate the sizing, weights, and flight characteristics of the aircraft concept. This model incorporates a vortex lattice model for aerodynamic analysis and utilizes simple beam analysis for structural sizing to evaluate performance over five mission configurations and 26 flight conditions.^{17,21} Key outputs include lift and drag data, structural and operating weights, and stability characteristics of the aircraft.

Financial Model

The financial model is based on empirical models developed by Markish¹⁵ and Markish and Willcox. Revenue, demand, and cost trends were fit to historical aircraft data, and the resulting equations provide price, baseline demand, and cost estimates using the outputs of the performance model.

Table 1 Best-fit parameters for empirical price in Eq. (3)¹⁶

Parameter	Narrow-body	Wide-body
k_1	0.735	0.508
k_2	0.427	0.697
α	1.910	2.760
$N_{\text{seats_ref}}$	419	419
Range_ref	8810 nmi	8810 nmi
Price_ref	\$148.7 M	\$148.7 M

Price is calculated as a function of range, number of passengers, and an operating cost adjustment as according to the following empirically derived model:

$$\text{Price} = \left[k_1 \times \left(\frac{N_{\text{seats}}}{N_{\text{seats_ref}}} \right)^\alpha + k_2 \times \left(\frac{\text{Range}}{\text{Range_ref}} \right) \right] \times \text{Price_ref} - \Delta \text{LC} \quad (3)$$

Range and number of seats N_{seats} are normalized by reference values (Range_ref, $N_{\text{seats_ref}}$) so that the entire value can be scaled by a reference price, Price_ref. These reference parameters, as well as k_1 , k_2 , and α , were determined through regression analysis based on public domain aircraft sales data.^{22,23} The resulting values are summarized in Table 1. The ΔLC parameter is a life-cycle cost adjustment based on fuel burn as a percentage of cash airplane-related operating cost (CAROC). Its value accounts for differences in the efficiencies of competing aircraft designs and reflects the idea that the more (less) efficient an airliner is to operate, the higher (lower) the price an airline is willing pay to own it.

Demand modeling approaches such as discrete choice analysis are used in the automotive industry. These methods focus on the response of the consumer to changes in product attributes. For the case of large commercial aircraft considered here, demand models are typically constructed using market forecasts. To determine demand, the design is classified as wide-body or narrow-body. A baseline demand quantity is then determined using market forecasts by passenger count, assuming a 50% market share for either of the two major airframe manufacturers. In reality, the performance of the aircraft would affect its demand, and this market-share assumption introduces further uncertainty in addition to that inherent in the market forecasts. Uncertainty in demand is accounted for using a probabilistic approach: actual aircraft demand is assumed to evolve stochastically from the baseline. A set of demand states is established using a geometric Brownian motion (GBM) model. This approach is commonly used in real options applications to model stochastically evolving processes. Although the actual commercial aircraft market dynamics would not strictly follow GBM and other approaches, such as using a mean-reverting process, are possible, this model has been shown to provide a satisfactory approximation.

According to the GBM model, demand uncertainty is accounted for using a Weiner process with rate of return $\mu = r_f$ and volatility σ , as measured from empirical commercial aircraft data.¹⁵ For a given baseline demand x_0 , the demand in the next time period would be $x_0 u$ with probability p , or $x_0 d$ with probability $1 - p$ according to the following equations^{24,25}:

$$p = (e^{r_f \Delta t} - d)/(u - d) \quad (4)$$

$$u = e^{\sigma \sqrt{\Delta t}} \quad (5)$$

$$d = 1/u \quad (6)$$

A key advantage of capturing the market uncertainty explicitly in Eq. (5) is the ability to discount at a known risk-free rate in Eq. (4). The probabilities in Eq. (4) are used to compute the expectation in Eq. (2). Values for the parameters r_f , σ , and Δt were determined from historical data¹⁵ and are summarized in Table 2.

Costs are estimated from the weight breakdown of the aircraft generated from the sizing model. A simple weight-based cost model

Table 2 Parameters for stochastic demand model¹⁵

Parameter	Narrow-body	Wide-body
σ	42.7%	45.6%
Program length	30 yr	30 yr
Δt	1 yr	1 yr
r_f	5.5%	5.5%
Inflation	1.2%	1.2%

is used because it can be expected to capture recurring and nonrecurring cost behavior at a level appropriate for the resolution of the aircraft representation in the optimization framework. This modeling approach has a number of limitations, including the underlying assumption that manufacturing cost scales with aircraft weight, which is clearly not necessarily the case. The model is described in more detail in Refs. 15 and 18.

The overall weight of the aircraft is divided according to part categories (e.g., wing, fuselage, etc.), which are assigned costs per pound from empirical data based on both the part and process (e.g., labor, materials, etc.) type. The total costs are then calculated by multiplying each weight by its relevant costs per pound and summing over all parts and processes. Estimates are provided for both nonrecurring and recurring costs. Nonrecurring costs are modeled as being incurred over the development timeframe as an approximate β -distribution based on empirical data. Recurring costs take into account a learning curve effect, such that the costs of manufacturing additional aircraft would be reduced over time.

Value-Based Design Optimization

The value-based design optimization is demonstrated for a notional BWB program. The baseline design is a BWB concept with a range of 7800 nmi and passenger capacity of 475, optimized for minimum gross take-off weight (GTOW). Initial demand for this design is 13.5 aircraft per year. Value-based optimization is carried out by replacing the GTOW objective function with E[NPV] as the final step in the optimization sequence, after the design is sufficiently defined to ensure convergence of the optimization algorithm and satisfaction of the design constraints. The resulting design is referred to as "E[NPV] optimal." The optimization uses several hundred design variables that describe the aircraft layout, including the payload geometry, control surface deflections, and spanwise distributions of wing twist, chord length, and incidence angle. The design variables, constraints, and optimization problem setup are described in detail in Ref. 17.

Different Objective Functions Result in Different Designs

Table 3 summarizes key differences between the baseline and E[NPV]-optimal designs, where changes are given relative to the baseline design. It is interesting to note that a noticeable increase in E[NPV] can be gained for little change in the GTOW. Further, the GTOW is seen to actually decrease by a very small margin (0.01%), suggesting that the baseline design may not have been fully converged on a minimum-GTOW solution. Closer inspection of the E[NPV]-optimal design explains these findings and demonstrates that the design has actually moved to a nearby, but different, place in the design space. The detailed breakdown of the layout and weight in Table 3 shows that the E[NPV]-optimal design trades aerodynamic performance and fuel efficiency for a lower structural empty weight and consequently lower GTOW and reduced manufacturing costs. A relatively large percentage change is observed for the chord length at the wing tip, although the actual length change is small. This reduction is driven by the high manufacturing and nonrecurring cost per pound associated with the winglet.

As propagated through the financial models, this resulted in a lower price because of the life-cycle cost correction for increased fuel consumption in Eq. (3), but also lower cost, hence a greater expected value. Despite the appearance of little change in the GTOW, the relative proportion of structural and fuel weights shifted, ultimately decreasing the overall GTOW slightly and improving the

Table 3 Percentage changes in E[NPV]-optimal design relative to baseline design

Parameter	Change, %
GTOW	-0.01
OEW	-0.45
Structural wt.	-1.1
Fuel wt.	+0.51
Gross area	+2.2
Aspect ratio	-2.2
Leading edge sweep	-0.85
Chord	
Root	+0.30
Tip	-23
Cruise L/D	-0.95
Cruise C_L	+1.2
E[NPV]	+2.3
Unit price	-0.14
Unit cost	-0.59

Table 4 Comparison of designs using E[NPV] vs traditional NPV as optimization objectives, expressed as percent increase or decrease relative to E[NPV]-optimal design

Parameter	Change, %	
	$r_d = 12\%$	$r_d = 20\%$
GTOW	+0.04	+0.26
OEW	-0.36	-0.63
Structural wt.	-0.85	-1.5
Fuel wt.	+0.53	+1.4
Cruise L/D	-0.46	-1.2
Cruise C_L	-1.2	-1.0
NPV	-154	-195
E[NPV]	-0.58	-3.7
Unit price	-0.42	-3.9
Unit cost	-0.46	-0.78
Computation time	-90	-90

Note: Designs resulting from optimization based on NPV found using Eq. (1) at the listed discount rate were subsequently evaluated using Eq. (2) to find E[NPV].

expected value. This captures the idea that changing the objective function will change the design, in this case because the sensitivity of E[NPV] is greater than that of the GTOW in this part of the design²⁶ space.

The changes shown in Table 3 are small, particularly when considered in the context of the relatively low fidelity of the underlying models. Thus, the specific numerical results are perhaps less relevant than the general observed trends in trade-off between aerodynamic performance, fuel efficiency, and manufacturing costs. Moreover, the calculated increase in program value of 2.3% is small, but, in an industry where profit margins are extremely tight, even a small difference could have a significant effect on the decision of whether to proceed with a program.

Stochastic E[NPV] vs Deterministic NPV

An important question to address is whether similar improvements in program value could be achieved by using a traditional deterministic value metric as the objective function. The BWB value-based optimization was repeated using the deterministic NPV, defined in Eq. (1), as the objective function. For the NPV metric, it is necessary to select a risk-adjusted discount rate. Two cases were considered: $r_d = 12\%$ and $r_d = 20\%$. A comparison between the resulting NPV-optimal designs and the E[NPV]-optimal design found previously is summarized in Table 4.

The comparison of these three designs shows two important issues with the deterministic value calculation. First, deterministic NPV is not an appropriate metric for assessing the profitability of an aircraft program, which has a long time span and a significant amount of uncertainty associated with market conditions. The NPV values corresponding to the designs in Table 4 are pessimistic—both because the value of flexibility was not considered and also

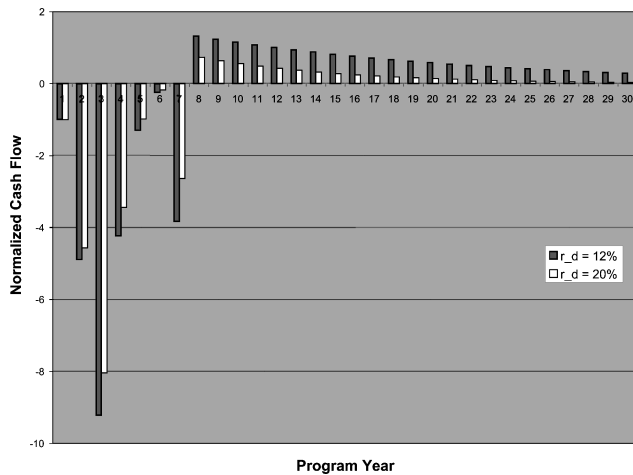


Fig. 2 Series of cash flows for program designs optimized using deterministic NPV at discount rates of 12% and 20%. Heavier discounting causes later profits to affect NPV less, causing the initial costs incurred to drive the value optimization. Values are normalized by the cash flow in year 1.

because use of a high discount rate to capture risk causes profits in later program years to carry very little weight. Moreover, the NPV result is highly dependent on the assumed value of r_d .

A second, separate issue—specific to the use of a value-based function for optimization—is that deterministic NPV is not an appropriate metric to use as an objective function. In the context of optimization, NPV is not a good indicator of favorable directions in which to move within the aircraft design space. Although the deterministic NPV estimates found previously were poor representations of the program value, the computational cost of optimizing the design using NPV rather than E[NPV] was reduced by 90%. This reduction in time represents a significant change from requiring on the order of days for an optimization run with E[NPV] to requiring only 2–3 h with NPV as the objective function. One could therefore conceive of using the efficient, deterministic calculation as an objective function in the optimization process and then subsequently evaluating the resulting NPV-optimal designs using the more expensive stochastic method. However, the results in Table 4 show clearly that this is not a viable approach, because the resulting NPV-optimal designs have substantially reduced value.

These differences are directly related to the choice of objective and its effect on the optimization progress. At both discount rates, the resulting designs gave up aerodynamic efficiency and lower fuel consumption to decrease structural weight. Although the marginal cost of each unit for the resulting designs was lower than that of the E[NPV]-optimal design, the prices were similarly lower because of the life-cycle cost adjustment in Eq. (3). At the higher discount rate (20%), the initial costs incurred have a greater effect on the overall value of the program, because later profits have a negligible effect due to discounting. This phenomenon is illustrated in Fig. 2, which shows the relative annual discounted cash flows for each NPV-optimal design. As a result, the optimization seeks to reduce the impact of development costs by reducing the structural weight. For a higher discount rate, this effect is magnified, resulting in a less favorable valuation. This significant difference suggests that E[NPV] is the better choice of design objective given the inherent difficulty in choosing an appropriate discount rate. In addition, this result demonstrates the danger of making design decisions using a conventional NPV metric, which tends to focus overly on the short term.

Setting Design Requirements

A key attribute of the value-based design methodology is the integrated consideration of technical and programmatic decisions. This enables the methodology to be used in determining how to best set program requirements and performance specifications. These could

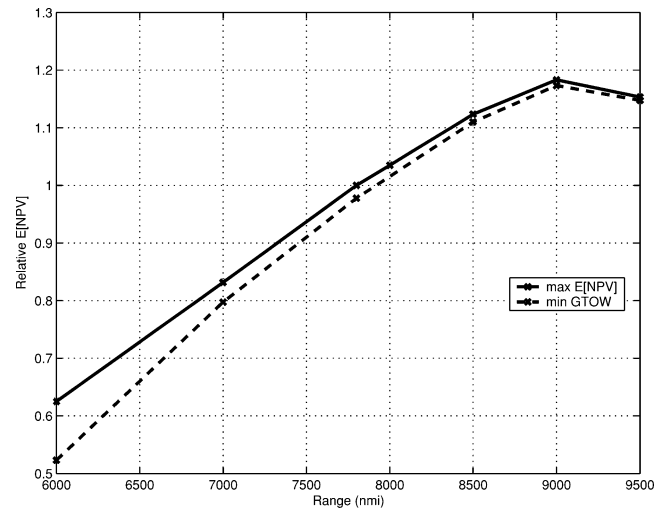


Fig. 3 Comparison of E[NPV] trends for performance- and value-based optimizations as a function of range, normalized by the E[NPV]-optimal design value.

include managerial decisions, such as how to best proceed with a program given particular market conditions, or technical decisions to set design performance requirements. Examples are presented of the design trade-offs inherent in setting the range and speed requirements for the BWB using the value-based approach.

Range vs E[NPV]

Adjusting the performance range of a new aircraft may be a necessary step in the design or marketing processes. Such a case is exemplified by the 7E7 program where Boeing offered a modified shorter-range version along with the initially planned long-range design to meet the needs of the launch customer.²⁷ Such a decision clearly has an effect on both the technical and financial characteristics of the design.

The BWB concept, which had a baseline range of 7800 nmi, was reoptimized for range settings varying from 6000 nmi to 9500 nmi. The resulting trend for E[NPV] is shown in Fig. 3. As the range increases, the GTOW increases because of the need for more fuel and corresponding structural growth. The figure shows that E[NPV] grows almost linearly between 6000 and 8500 nmi. For higher ranges, the slope begins to decrease. Finally, the trend reverses, and the 9500-nmi design is actually less profitable than the 9000-nmi design. This behavior is a function of the financial models, notably Eq. (3) for price, which varies linearly with range. At a sufficiently large range, however, the life-cycle cost adjustment for the necessary increase in fuel consumption outweighs the benefit of longer range. It was assumed for this analysis that the baseline demand, which is modeled as a function of passenger capacity, remains constant.

Comparison to designs optimized for performance over the same set of ranges, as seen in Fig. 3, illustrates the benefits of using value as the design objective in addition to simply evaluating the design based on a combination of performance and financial analysis. At longer ranges, there is less disparity between performance- and financial-optimal designs, but at lower ranges the discrepancies become increasingly wider. Specifically, the E[NPV]-optimal (7800-nmi) design represents a 2.3% improvement in E[NPV] over the baseline; at the lower 6000-nmi range, the design optimized for value has an E[NPV] 19% greater than the corresponding performance-optimal design.

Speed vs E[NPV]

An analogous study can be performed with respect to the cruise speed to determine how to best set a cruise Mach number to maximize the value of the program. Again, the importance of such analysis is made clear by the 7E7 example and the need to weigh the value of speed vs other performance concerns in choosing to develop the 7E7 rather than the Sonic Cruiser. Figure 4 shows the

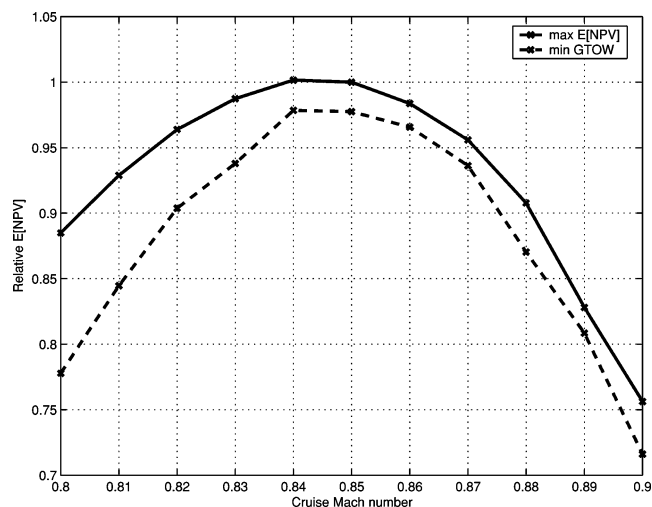


Fig. 4 Comparison of E[NPV] trends for performance- and value-based optimizations as a function of cruise Mach number, normalized by the E[NPV]-optimal design value.

resulting trend for a sweep of Mach settings from 0.8 to 0.9 for the conditions related to the cruise portion of the mission profile, as described in Wakayama.¹⁷ The baseline speed, 0.85 M , is very close to the actual best-case speed of 0.84 M , with only a 0.16% difference in E[NPV]. Optimizing the BWB design for both higher and lower cruise Mach settings is less beneficial. At the lower end of the speeds evaluated, setting the cruise requirement to 0.8 M results in an E[NPV] 12% worse than the E[NPV]-optimal solution. For higher speeds, the trend is even more detrimental, with the 0.9- M design having a program value 24% worse than the optimum.

The model did not account for changes in specific fuel consumption (SFC) due to the varying design speeds. This would affect the trend in Fig. 4, because the SFC would be lower at lower Mach settings and higher in the opposite case, translating to lower GTOW around 0.8 M and even higher GTOW around 0.9 M . An additional limitation of this study is that the life-cycle cost adjustment ΔLC in Eq. (3) accounts for the operating cost effects of increased fuel consumption but not the utilization benefits of faster speeds. Shorter travel times resulting from higher cruise speeds would allow the airline operator to use the aircraft more often, the appeal of which could be accounted for by a price increase analogous to the fuel-burn effect. It is unclear to what extent such an adjustment would offset the effects of varying the SFC, so the trends shown in Fig. 4 could remain similar. Such an adjustment to account for utilization in the price would be a useful extension of the financial models. These limitations of the models restrict the validity of specific quantitative conclusions that can be drawn from the results in Fig. 4; however, as discussed in the following paragraphs, a more detailed analysis of these results provides useful insight to the value of speed and the corresponding design trades.

As with the range study, the designs optimized for maximum E[NPV] are more profitable than performance-only designs for minimum GTOW across all speeds considered. This result is depicted in Fig. 4. At lower speeds, the gap between the two widens, and E[NPV] for the minimum-GTOW 0.8- M design is 12% worse than the corresponding maximum-E[NPV] solution. For higher speeds, the disparity is smaller, with a 5.3% decrease in E[NPV] from the 0.9- M value-optimal design to the performance-optimal equivalent. As was the case at shorter ranges in the previous section, the ability of the new objective to influence the design seems to be more important at lower speeds.

A breakdown of the performance-only and financial optima at 0.8 M illustrates why the difference in E[NPV] is more noticeable. The value-optimal design shows a relatively small decrease in structural weight; most of the difference in E[NPV] is made up by the higher price of the maximum-E[NPV] solution due to improved fuel efficiency. This result is interesting in that the optimization drove

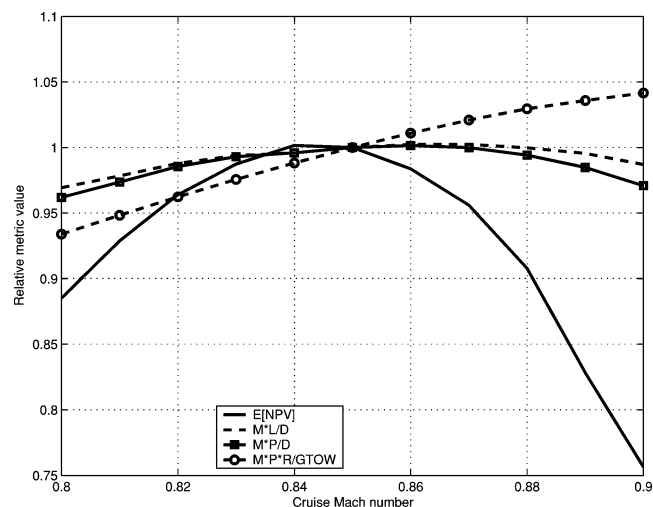


Fig. 5 Trends for design performance and value metrics as a function of cruise Mach number, normalized by their respective baseline (0.85 M) values.

the design to decrease the fuel consumption as opposed to decreasing the structural weight, as seen in previous comparisons between optimizations minimizing GTOW vs maximizing E[NPV].

Figure 4 indicates that Mach 0.84 is the optimal cruise speed for using E[NPV] as the figure of merit. Several other traditional metrics that relate the performance of the design to its marketability could be evaluated instead to estimate the optimum cruise speed. Figure 5 presents the results for E[NPV] and three other such metrics for the range of Mach numbers considered previously, normalized by their value at the baseline cruise Mach (0.85). The additional figures of merit are as follows:

$M \times L/D$ – Mach number multiplied by lift-to-drag ratio;

$(M \times P)/D$ – Mach number multiplied by payload weight, divided by drag; and

$(M \times P \times R)/GTOW$ – Mach number multiplied by payload and range, divided by GTOW.

The first metric is derived from the Breguet range equation for constant SFC and weights, whereas the latter two focus on the traditional design merits of increased payload and range, traded against decreased GTOW. Emphasizing only speed and L/D , 0.87 M is the new optimum, reflecting better aerodynamic performance at a slightly higher cruise speed. Taking into account payload capacity instead of overall lift reduces that value slightly, to 0.86 M , recognizing that extra speed does not increase capacity as it does drag. Finally, introducing range and the traditional design metric, GTOW, in addition to capacity shifts the trend dramatically, placing far more value on speed, with an optimum of Mach 0.9 or possibly higher if the trend were extrapolated. In this case, the value of added speed is not fully offset by consequent increases in GTOW, allowing the metric to grow with higher Mach numbers for fixed payload weight and range.

None of these design metrics explicitly estimate the profitability of a program, however, and E[NPV] remains the most complete figure of merit for evaluating the contributions of the individual parameters featured previously. Despite potential limitations of the financial models in their ability to represent the value of cruise Mach number, this case study further illustrates the usefulness of a coupled performance and financial approach to design. It provides a new framework for setting program requirements and, specifically, for understanding the effect of cruise speed on profitability.

Quantitative Assessment of Business Risk

In the MDO design examples illustrated previously, some design uncertainty was accounted for, specifically the demand volatility. In practice, many additional forms of uncertainty will arise, including technical uncertainty related to the design performance and other forms of financial or market uncertainty. It is desirable to understand

Table 5 Sensitivities of E[NPV]-optimal design to uncertainties in technical and financial parameters

Parameter varied	Related parameter	E[NPV] change, % (per 1% parameter increase)
Nonrecurring cost	OEW	-0.71
SFC	Price	-10
Fuel price	—	-6.6
Demand		
Volatility	—	-2.8
Initial	—	+1.4
Recurring cost (LRMC)	—	-3.8

the effect of potential variability, as well as the related risk faced by the program, by quantifying the effect on program value. The stochastic methodology presented here allows this assessment to be carried out in a systematic manner.

Relative Program Risk Assessment

Table 5 presents an overview of sensitivity analyses of the baseline design with respect to both financial and technical uncertainties. Changes in SFC, demand, fuel price, and recurring costs were assessed directly, whereas changes in OEW were related to an increase in nonrecurring cost as described in the next subsection. From the table, it can be seen that the design is most sensitive to fuel efficiency, and by extension, price. The next largest effect on E[NPV] is due to uncertainty in fuel price, followed by recurring costs and then the demand parameters. These results show that changes to the long-term program cash flows are the leading source of business risk.

The fuel-price sensitivity in Table 5 can be seen to be substantially greater than that of the other financial parameters but less important than the sensitivity to changes in SFC. Intuitively, this result makes sense, because an increase in SFC would affect both the aircraft operating efficiency and the overall weight of the design. With an increase in fuel price of approximately 25%, the baseline design becomes unprofitable. If the aircraft were redesigned under less favorable fuel-price assumptions, we would expect to see significantly different results in the optimal balance between aerodynamic efficiency and structural weight. The fuel-price-sensitivity result demonstrates the importance of market assumptions in conceptual aircraft design and again emphasizes the value of an integrated technical/financial design process.

The relatively low sensitivity of E[NPV] to nonrecurring cost compared to long-run marginal cost (LRMC) and price suggests that by correcting problems with the design in the development stage, more of the baseline value can be retained than would be the case for a design that does not meet its intended performance. Although the idea that spending more money earlier to save money on a better design later is the suggested strategy to mitigate business risk for the design example considered, it is important to note that a deterministic valuation would rule out such a strategy. As seen in the previous value-based-optimization results, a deterministic valuation drives the reduction of up-front costs, because later cash flows have less effect on overall profitability because of heavy discounting.

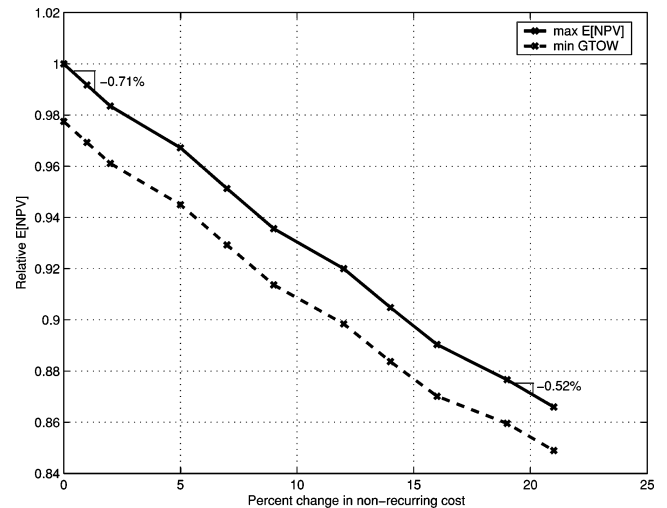
Technical Uncertainty

Technical uncertainty could exist in many forms, many of which result in aircraft weight growth—hence the basis for the traditional design goal of minimum GTOW. There are essentially three scenarios that may evolve as the program moves ahead with the design.

1) The aircraft is sold at the higher weight to be used at less than its maximum (intended) range or with added fuel volume if possible to achieve its intended range. The aircraft may meet the operators' needs but will be more expensive to operate. This scenario results in higher manufacturing costs, a lower price, or both.

2) The aircraft is redesigned or weight is eliminated to meet the original specifications, resulting in higher nonrecurring development costs.

3) The aircraft is unable to meet its performance guarantees, either because of outside sources or failed redesign. Additional non-

**Fig. 6** Relative E[NPV] for max-value and min-GTOW designs vs percent change in nonrecurring cost, normalized by max-value E[NPV].

recurring costs may be incurred from attempts to fix the problem, and recurring costs will be higher despite a lower price because of missed performance goals.

In scenario 2, technical uncertainty leading to an increase in operating empty weight (OEW) can be translated to an increase in program nonrecurring cost. Increases in nonrecurring cost up to approximately 20% of the baseline design were examined to find the related effect on E[NPV]; no decreases in development costs were considered, because a lower-than-expected OEW would not necessarily translate into cost savings. The change in nonrecurring cost was added in the final time period of the development stages to represent reengineering after a significant amount had already been invested in design and capital costs.

Figure 6 shows the results of this sensitivity analysis for maximum-E[NPV] and minimum-GTOW designs, normalized by the baseline value optimum. The value-optimized design resulted in higher E[NPV] for all changes in nonrecurring cost than the performance-only design. Higher incurred costs result in lower program E[NPV], and cost reductions increase value, which is unremarkable except that the sensitivity of E[NPV] to nonrecurring cost overruns is rather low. A 5% increase in development costs results in less than a 5% decrease in program value. Specifically, around the baseline design, the change in E[NPV] is 0.71% of the baseline value for every percent change in nonrecurring cost, as seen on the plot. As the design becomes more unfavorable, the real options approach dictates that proceeding with the program becomes less likely and its value decreases, but its expected value will never become negative.

Financial Uncertainty

In addition to the technical uncertainty inherent in an design program, the market for aircraft and general economic conditions can introduce separate financial uncertainties. These could relate to the nonrecurring or recurring costs, price, or demand.

Demand Volatility

The stochastic demand model accounts for demand uncertainty in terms of the volatility of the aircraft market. This volatility is based upon empirical data, however, and subject to uncertainty itself. Variance in the volatility represents a market risk that could affect program value. A sensitivity analysis performed on the design by varying the demand volatility σ can help assess the level of risk faced as a result of shifting demand over the program lifetime as propagated through the stochastic demand model. The volatility was varied $\pm 15\%$ from its baseline value of 45.6%, which represents the average volatility for a wide-body aircraft. Values for σ within a range of $\pm 10\%$ from the aggregate volatility for all wide-body aircraft¹⁵ were also examined.

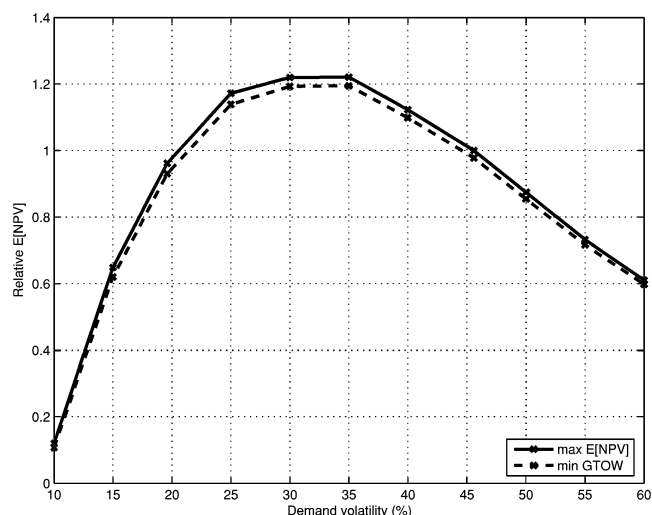


Fig. 7 Relative E[NPV] for max-value and min-GTOW designs vs demand volatility, normalized by max-value E[NPV].

For each σ , E[NPV] results were found for both the maximum-E[NPV] and minimum-GTOW designs. Figure 7 compares the trends for the two designs at varying volatilities, with E[NPV] normalized by the optimum value at the baseline, $\sigma = 45.6\%$. In all cases, the E[NPV]-optimal solution shows improved value over the performance-only solution. Because of its improved profitability, the maximum-value solution will provide value in both the case of fairly constant demand (low volatility) and at demand extremes in the case of higher volatility.

The relationship between E[NPV] and demand volatility for both design solutions follows an expected pattern. At lower volatilities, near the aggregate wide-body value, the overall program value decreases as the possibility for demand growth beyond the initial quantity is lessened. Conversely, at higher volatilities beyond the baseline average wide-body value, E[NPV] also decreases as the probability of low future demand quantities increases. Volatilities between the aggregate and average data points result in an even better valuation than the original maximum-E[NPV] design, however, striking a balance between significant demand growth and the potential of markedly reduced demand. In a sense, these values for σ capture the idea of a high-risk, high-reward design where increased demand uncertainty is such that a confluence of beneficial events could result in a highly successful program, and the probability of a market decline is not high enough to drastically lower the expected value. No similar analysis can be performed for solutions optimized for deterministic NPV, because increased volatility would simply be accounted for in Eq. (1) by increasing the discount rate. The resulting trend would be unable to capture any of the value of increased risk, because constantly decreasing NPV with increasing r_d would replace the curve seen in Fig. 7.

Sensitivity analysis of the E[NPV]-optimal results correlates the percentage increase or decrease in E[NPV] due to a percentage point of volatility gained or lost. The most significant effects are seen at lower volatilities, where a 1% change as σ increases leads to a 9.2% increase in E[NPV] relative to the baseline value. At the other extreme, continually increasing volatility past the baseline of 45.6% results in an approximately 2.7% decrease in program value. Sensitivities are at their lowest in the region around the peak estimates for E[NPV], where a percent increase in σ from 30% causes less than a percent increase in program value, or 0.48%.

Recurring Costs

Variability in the long-run costs of aircraft production could be due to variations in material, labor, or other costs. Sensitivity analysis of E[NPV] given a change in the LRMC was performed by applying a multiplier to the baseline cost. Values ranged from -5% to $+15\%$ of the baseline LRMC to examine the case of slight cost reductions and the more likely scenario of significant cost over-

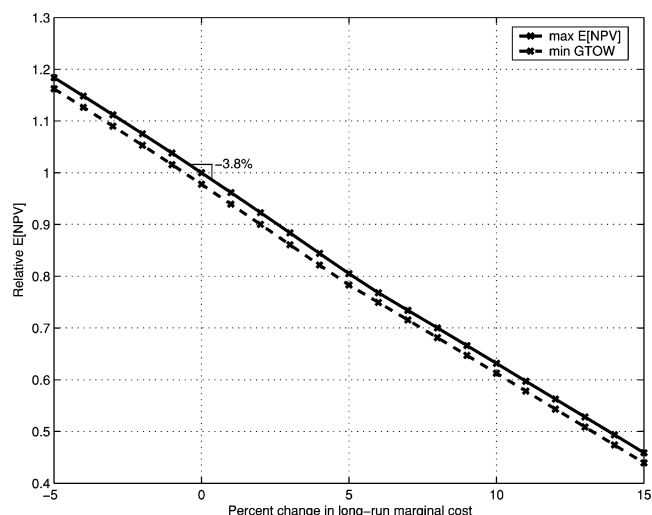


Fig. 8 Relative E[NPV] for max-value and min-GTOW designs vs percent change in LRMC, normalized by max-value E[NPV].

runs. The resulting trends for manufacturing cost variability for the E[NPV]- and GTOW-optimal designs are presented in Fig. 8, normalized by the baseline (zero change in LRMC) maximum-E[NPV] solution.

Although the design resulting from the optimization of E[NPV] retains more of its value given less favorable program conditions due to cost uncertainty than the performance-only optimum, it should still be noted that both designs lose approximately 20% of their value with only a 5% increase in LRMC. These sensitivities are much more significant than those relating the effect of nonrecurring cost uncertainty to E[NPV], indicating that eliminating design uncertainty earlier is more beneficial than producing a suboptimal design—despite the larger impact of incurring the cost of design changes earlier—due to heavier discounting of later cash flows.

The results in this section demonstrate a quantitative assessment of business risk using sensitivity analyses of key technical and financial parameters. Ideally, the decision-making framework should incorporate these uncertainties directly into the optimization. One approach would be to include stochastic models for these parameters in the DP formulation, using a process similar to that described here for aircraft demand. This approach is unfortunately limited by the “curse of dimensionality,” because each stochastic parameter would require an additional state variable in Eq. (2). Other approaches being explored for design optimization under uncertainty, which remains a challenging and open research area, include game theory and decision-based design.

Conclusions

A stochastic value-based methodology for conceptual aircraft design has been presented. This methodology builds on previous research in aircraft valuation by combining technical design and value in an aircraft conceptual-design optimization framework. The framework helps to bridge the gap between technical design and financial analysis, specifically by helping engineers and managers to better understand the financial implications of design decisions, including consideration of program risks. Although state-of-the-art valuation methodologies may be used at aircraft companies by financial analysts to value future aircraft programs, such methods are not directly integrated with the technical design process.

The results of a case study performed using the BWB concept show that improvements in the expected NPV of a program can be gained by incorporating financial models into a design-optimization framework. Changes in the value-optimal design compared to the baseline minimum-GTOW configuration demonstrate that changing the objective in the optimization results in a different design and that the optimizer chooses to trade aerodynamic performance and thus aircraft price for reduced structural weight and thus aircraft cost. For this case, the observed design changes are small, particularly

given the relatively low fidelity of the underlying models; however, the general trends exhibited by the results provide useful insight to the trade-offs between performance and cost.

Deterministic NPV was shown to be an unsuitable objective function for value-based optimization. An inaccurate assessment of the value of a design is provided, and, more important, the optimizer is driven to undesirable places in the design space. In particular, the effect of the arbitrary choice of risk-adjusted discount rate has a large effect on the resulting design by causing design decisions to focus overly on reducing short-term development costs.

Examples of the application of value-based MDO incorporate value into the design process for setting range and speed requirements. Subsequent sensitivity analyses of a value-optimal design allow quantification of the relative business risks associated with uncertainty in individual technical and financial parameters. Although specific quantitative conclusions are highly dependent on the case at hand, the underlying models, and the assumptions, the following trends are highlighted:

First, a value-based approach to MDO allows more fully informed program decisions regarding design specifications, as evidenced by the findings that longer ranges and higher speeds offer diminishing returns in value—results not immediately obvious from performance-only analysis.

Second, sensitivity analyses indicate that the effects of fuel cost, recurring cost, and aircraft price on the long-term profitability of the design pose the greatest risk. Market uncertainty is also a source of considerable risk; however, the stochastic valuation is better able to account for the possibility to take advantage of so-called high-risk, high-reward situations.

Third, E[NPV] shows that incurring costs early in a program to ensure a successful design represents a safer strategy than going to market with a design that has missed performance goals. By contrast, deterministic NPV continually deemphasizes the importance of long-term profitability.

Finally, stochastic E[NPV] is again demonstrated to be an improved valuation metric over deterministic NPV. It models both the ability to take advantage of favorable program variability as well as to mitigate the effect on program value given an unfavorable design or market.

The history of aircraft MDO originated in aero-structural optimization and has proceeded to encompass many other important aspects of aircraft systems design. However, Sobieszczanski-Sobieski and Haftka²⁸ note, “there are still very few instances in which the aerospace vehicle systems are optimized for their total performance, including cost as one of the important metrics of such performance.” Introducing stochastic value to measure cost attempts to fill this niche and complement the similar work done by others to the same end, and can be viewed as a new and necessary step toward truly optimized aircraft.

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