Value-Based Multidisciplinary Optimization for Commercial Aircraft Design

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ABSTRACT

Traditional commercial aircraft design attempts to achieve improved performance and reduced operating costs by minimizing maximum takeoff weight. From the point of view of an aircraft manufacturer, however, this method does not guarantee the financial viability of an aircraft program. A better design approach would take into account not only aircraft performance but also factors such as aircraft demand, market uncertainty, and development and manufacturing cost. This paper outlines a design method resulting in an optimization framework to consider both performance and finance in aircraft program design. The optimization procedure couples a simplified aircraft performance model with a program valuation technique based on real options theory to address uncertain market demand. This new methodology is then applied to an aircraft design example. Results include comparison of performance- and financial-optimal designs, as well as sensitivity analyses to quantify the effects of technical uncertainty on business risk.

INTRODUCTION

The historical choice of minimizing gross take-off weight (GTOW) as the objective in aircraft design is intended to improve performance and subsequently lower operating costs, primarily through reduced fuel consumption. However, such an approach does not guarantee the profitability of a given aircraft design from the perspective of the airframe manufacturer. In an increasingly competitive market for commercial aircraft, manufacturers may wish to design for improved financial viability of an aircraft program, as well as technical merit, before undertaking such a costly investment.

To assess the long-term financial impact of an aircraft program, a value-based approach is recommended. Such an approach might still account for performance while also incorporating the following elements to assess predicted cash flows into and out of the program: manufacturing and development costs; product demand; operating cost to the customer; and market factors, such as competition, uncertainty, and expected growth. A valuation methodology has been proposed that draws on financial options theory and uses dynamic programming to estimate the costs, price, and demand associated with a previously optimized aircraft design.[1] By accounting for market uncertainty and addressing the idea of managerial flexibility, this methodology allows the user to calculate an optimal value for an aircraft program. Further, quantifying market uncertainty directly allows for a more explicit accounting of perceived program risk, as opposed to traditional valuation techniques that rely on an assumed discount rate on future cash flows generated by a program. The valuation technique is detailed in the next section, "Program Valuation."

In this paper, coupled performance/financial design is effected by incorporating the valuation methodology into the design optimization process, as shown in Figure 1 (fig. 2). Using simple financial models, a framework has been created that couples a higher-order performance model with financial estimation tools and an algorithm for computing expected program value. A single program concept incorporating technical design as well as financial parameters can then be optimized in terms of specific performance or business goals, e.g., minimizing GTOW or maximizing program value. These individual modules are described, along with the overall optimization framework, in the third and fourth sections: "Simulation Model" and "Optimization Framework."

The model was then used to solve for financially optimal designs of a single aircraft, a Boeing 777-class airliner. The number of passengers and aircraft range were chosen to make up the design vector. Aircraft manufacturers and airlines alike often view the basis of value in commercial aircraft in terms of range and carrying capacity when making a decision on what kind of aircraft to manufacture or purchase. Single-objective optimization was carried out using program value, as measured in terms of expected net present value (NPV), as the objective to be maximized. Comparisons may then be made to the actual design of the 777 to identify the potential tradespace between performance- and value-based optimization,

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as well as to the results of using traditional valuation techniques. Sensitivity analyses may also be performed to consider the effects of uncertainty in various technical parameters on the NPV outcome – in a sense, quantifying financial risk. Preliminary findings are summarized in the "Optimization Results" section.

PROGRAM VALUATION

By choosing maximum program value as a design objective, many of the above factors – performance, cost, demand – can be considered by modeling their effects on overall cost and revenue. A properly chosen value calculation based on these cash flows can effectively capture the entire program design in a single number. The value metric chosen to model the financial viability of an aircraft program is NPV, which is a figure commonly used to assess aircraft profitability. NPV may be used to estimate the future value of an investment or other fiscal activity in terms of current money. This metric is especially useful in that it accounts for the time value of money and provides a clear estimate of an investment's future value.

The calculation of NPV is traditionally accomplished by estimating a risk-adjusted discount factor to account for the opportunity cost of capital and the perceived risk inherent in a venture. The value itself is found through the summation of future cash flows discounted by the risk-adjusted rate. However, this approach is limited in some respects in its ability to provide such a definitive valuation. Finding or calculating a discount rate that includes the effects of risk may be a difficult, and ultimately somewhat arbitrary, process – resulting in a less accurate assessment of risk. Further, the value of flexibility or decision-making at a future time cannot be accounted for in such a straightforward calculation of NPV.

The real options valuation approach attempts to address these flaws. Using a dynamic programming formulation, the optimal expected NPV can be found for an aircraft program, given uncertain market conditions. The problem takes as states the program mode, including design, tooling, and production stages, and quantity demanded according to a stochastic model. The manufacturer may choose to pause or cancel the program when in certain modes, effectively investing no more money if future market conditions appear unfavorable. Then a series of optimal cash flows and program decisions can be found which will lead to a maximum expected value for NPV. A key advantage to this approach is that explicitly including a model for demand uncertainty and applying real options theory allows discounting according to the risk-free rate instead of an assumed discount rate, and thus a more explicit accounting for the perceived risk of a venture.

SIMULATION MODEL

The dynamic programming problem above requires annual cost and revenue estimates for the aircraft program. These values are derived from models that take both the design vector and resulting aircraft design parameters as inputs. The simulation model first uses the design vector to generate first-order size and performance estimates for an aircraft concept. Then, relevant design values are used by the financial modules to calculate cost, price, and baseline demand for the resulting aircraft program. Finally, expected NPV is determined from the cost, revenue, and stochastic demand model outputs according to the dynamic programming algorithm. These modules are described in further detail below.

Physical Model

The physical aircraft model was based on an aircraft sizing routine developed by Liebeck.[2] This routine was intended to generate aircraft size and performance characteristics that meet design specifications set by the user. The equations found in the routine were derived from several different sources, including aerodynamic first principles, empirical aircraft data, and approximations and rules of thumb from experience in the aviation industry. The physical model was composed of three main sub-modules: aerodynamics, weights, and performance. Inputs were the overall design vector – number of passengers and aircraft range – in addition to a number of used-specified parameters. Wing shape, cruise conditions, engine configuration, and takeoff and landing performance were fixed to values approximating a 777-class aircraft before optimization runs were begun.

The aerodynamics module assumed an initial cruise lift coefficient and calculated preliminary values for the thickness-to-chord ratio and aircraft wing loading at takeoff, cruise, and landing conditions. A value for the fuel fraction of the aircraft was also determined from the engine properties and desired range. With this information, the aerodynamics module calculated the actual value of the lift coefficient at

initial cruise using first principles. The weights module used these results to calculate the thrust loading for a given engine, and wing loading. Fuselage dimensions were calculated based on the payload requirements and desired seating configuration. The module then developed a series of weight fractions for several main aircraft components, expressed as fractions of the GTOW of the aircraft. The as-yet-unknown GTOW was then solved for iteratively, and the component weights could be calculated. The performance module rounded out the design of the aircraft by determining several standard aircraft characteristics and dimensions. From the outputs of the sizing and weights models, it was possible to calculate drag coefficients for the wing, fuselage, and other aircraft components and arrive at a drag coefficient and lift-to-drag ratio for the whole aircraft.

Financial Model

The financial model (cost, revenue, and value modules) is based on empirical models developed in Markish. Simplified models were used in order to reduce computation time for the purposes of demonstrating the optimization framework. Cost and revenue parts were based largely on fitting trends to historical aircraft data. Resulting equations were provided to calculate actual cost, price, and baseline demand estimates using the design vector and other aircraft parameters.

Revenue. First, the design was classified in terms of its size (wide- or narrowbody, from a range threshold) and number of passengers. Each class corresponded to a demand "bucket" to determine a baseline quantity, assuming a 50% market share for either of the two major airframe manufacturers. Demand was then assumed to evolve stochastically, as described below. Price was calculated as a function of range, number of passengers, and an operating cost adjustment as follows.

$$Price = [k_1 \times (\frac{N\_seats}{N\_seats\_ref})^{\alpha} + k_2 \times (\frac{Range}{Range\_ref})] \times Price\_ref - \Delta LC \quad (1)$$

Range and number of seats were normalized by reference values so that the entire value could be scaled by a reference price. The ΔLC parameter was a lifecycle cost adjustment based on fuel burn as a percentage of Cash Airplane-Related Operating Cost (CAROC). Its value accounted for differences in the efficiencies of competing aircraft designs, and reflected the idea that the more (less) efficient an airliner is to operate, the higher (lower) the price an airline is willing pay to own it.

Cost. Costs were estimated from the weight breakdown of the aircraft generated from the sizing model. The overall weight was divided according to part categories (e.g., wing, fuselage, etc.), which were assigned costs per pound from empirical data based on both the part and process (e.g., labor, materials, etc.) type. The total costs could then be calculated by multiplying each weight by its relevant costs per pound and summing over all parts and processes. Estimates were provided for both non-recurring (e.g., engineering, tooling, etc.) and recurring (e.g., manufacturing, quality assurance, etc.) costs. Non-recurring costs were modeled as being incurred over the development timeframe as an approximate β-distribution based on empirical data. Recurring costs took into account a learning curve effect, such that the costs of manufacturing additional aircraft were reduced over time. A theoretical first unit (TFU) cost for the first production aircraft was calculated through the above process, and additional unit costs were discounted according to the following learning curve equation.

$$Y_n = Y_0 n^{(\ln b / \ln 2)} \quad (2)$$

Y_0 represented the TFU cost, Y_n was the cost of unit n, and b was the learning curve factor or slope. The distribution of recurring costs was then found by summing the costs of all aircraft produced each time period.

Stochastic Demand. From the previously determined baseline demand, a set of demand states was established using a geometric Brownian motion model. Uncertainty was modeled as a Weiner process with demand growth μ and volatility σ, as measured from empirical commercial aircraft data. As such, for a given baseline demand x_0, the demand in the next time period would be x_0u with a probability p, or x_0d with a probability 1-p according to the following equations,

$$p = \frac{e^{\mu \Delta t} - d}{u - d} \quad (3)$$

$$u = e^{\sigma \sqrt{\Delta t}}$$
$$d = 1/u \quad (4)$$

with μ as the risk-free rate of investment in this case. Using this model, a binomial lattice of demand states was constructed, as illustrated in Figure 1, as well as a matrix of corresponding transition probabilities. These demand states and probabilities were then used by the value module to calculate the optimal expected NPV using a dynamic programming algorithm.

Value. As described previously, value was measured in terms of expected NPV. This quantity was calculated using a dynamic programming algorithm taking current program mode and quantity demanded as states. The problem also accounted for uncertainty in terms of demand volatility. The objective function, expected NPV, was calculated by summing optimal periodic cash flows determined according to the Bellman equation.[3] The cash flow in a given period was the sum of sales profits, less the design and production costs incurred and cost of switching modes. Cash flows were further adjusted for inflation and discounted by the risk-free rate. Outputs were the expected NPV and a set of program "decision rules" that dictate the optimal conditions for design and production scheduling. The decision rules give, as a function of time and current market condition, the minimum demand to enter a particular program mode. This series of optimal decisions accounted for managerial flexibility to handle uncertain demand, as derived from the real options approach discussed previously.

Model Validation

The sizing, cost, and price models were calibrated with inputs corresponding to a Boeing 777-200ER, which had been used as a reference aircraft in setting up the financial models. The sizing model was found to produce a breakdown of weights representative of the 777, and also remained applicable for other large widebody aircraft (747, A340, etc.) but was less accurate for smaller such aircraft (767, A330). The cost model followed a similar pattern, as it was calibrated for a 777-type aircraft as well, and approximated costs based on the estimated weight breakdown. Sensitivity analysis of long-run marginal cost with respect to GTOW showed a linear relationship as expected. Finally, the price model provided good estimates for various widebody aircraft compared to available data. Further, the breakdown of price into ownership and CAROC components closely followed a typical 40% ownership and 60% CAROC split, and sensitivity analysis with respect to GTOW reflected a change in that breakdown accordingly. The simplified framework was deemed sufficient to proceed with further optimization runs and sensitivity analyses using inputs for a 777-class aircraft.

Outputs from the simplified value model were benchmarked against NPV estimates generated by Markish using a more complete implementation of the code. Sizing, price, and cost estimates were produced for three representative aircraft programs, with results for the expected NPV in each case summarized in Table 1 below. Discrepancies that arose were satisfactorily explained before proceeding to development of the overall optimization framework using this simulation model.

Table 1. Comparison of expected NPV from DFV code vs. simplified model using DFV price, cost estimates and WingMOD weight inputs.

	Original E[NPV] ($B)	**Simplified E[NPV] ($B)**
Aircraft 1	5.95	5.52
Aircraft 2	2.26	2.28
Aircraft 3	14.62	14.47

OPTIMIZATION FRAMEWORK

A design optimization framework coupled the performance and financial models with an optimization routine as illustrated in Figure 2. An initial design vector (number of passengers, range) was provided to the physical model to estimate aircraft sizing and performance characteristics. Its relevant outputs were then used by the cost and revenue models to approximate cost, price, and baseline demand figures for the design. The value model used the dynamic programming algorithm, which accounted for

market growth and uncertainty, to determine a set of optimal design decisions and the objective, expected NPV. To complete the loop, the objective was input to the optimization routine; the optimization algorithm determined a new design vector to attain the goal of maximized NPV; and the new variable values were passed to the simulation model once again to begin a new iteration. The optimizer represented in Figure 2 used a public domain adaptive simulated annealing (ASA) algorithm.[4] A heuristic optimization technique was preferred so that the entire design space would be searched for the optimal value despite discontinuities due to changes in baseline demand as a function of number of passengers, as illustrated by the plot of expected NPV versus number of passengers in Figure 3.

OPTIMIZATION RESULTS

Following model refinement and validation, a series of sensitivity analyses were performed investigating the effect of changes in GTOW on non-optimal solutions for NPV. These used inputs for range and number of passengers corresponding to a 777. The effect of changing GTOW on expected NPV is illustrated in Figure 4. The expected NPV approaches a limit of zero as GTOW increases, thus the effect of increasing weight is diminished over time, which may seem contrary to expectations. This is due to the real options approach implemented through the dynamic programming algorithm, which prevents the possibility of a negative expected NPV. Recall that the dynamic programming problem is solved to find an optimal decision-making strategy and the corresponding optimal expected NPV. One could always achieve an expected NPV of zero by following a strategy of "do nothing regardless of market conditions". Therefore, the optimal result cannot yield a negative expected NPV. Figure 5 contrasts the trend for expected NPV from Figure 4 with a deterministic NPV calculation that does not allow managerial flexibility, i.e. it is assumed that once the aircraft program begins, all phases (design, tooling, and production) are carried out, regardless of market conditions. This latter calculation is representative of the way in which aircraft programs are typically evaluated, and leads to linearly decreasing NPV with increased GTOW over the range of weight consideredThis plot demonstrates that using an assumed discount rate (?) and ignoring the value of managerial flexibility may severely undervalue a program.

The same sensitivity analysis was then considered with reduced annual demand volatility σ. The original value of 45.6% corresponded to the average volatility for a given type of widebody aircraft; the new value of 19.6% represented the average aggregate volatility of all widebody deliveries. NPV sensitivity to GTOW is compared for these two cases in Figure 6. Note that the expected NPV for the lower volatility is actually lower, as well. This indicates that a lower volatility may correspond to lower overall risk, but also a smaller optimal return on investment. Again, a traditional NPV calculation would likely account for higher volatility with an adjustment increasing the discount rate, so the possibility of a higher payoff resulting from the "riskier" venture would not be captured by such an approach.

Another consideration was the effect of increasing GTOW on the decision rules, which represent the threshold demand levels needed to move through the various modes of an aircraft program. The initial threshold (at program time = 0) is plotted in Figure 7 for the decisions to begin design, tooling, and production, as a function of increase in takeoff weight. Exponential curves were then fitted to the plots, since the demand is an exponential function of the volatility (Equation 4) in the geometric Brownian motion model used by the algorithm. However, the sensitivity to GTOW is very nearly linear, so finer resolution may be desired. It can be seen from the figure that an increase in GTOW of 5000 lbs causes the threshold demand to begin design to increase from X to Y, while the threshold to begin production increases from W to Z. The threshold for production is much lower than that for design, since the initial non-recurring investment has already been made. Such curves show how technical uncertainty interacts with market uncertainty and could be used to relate managerial decisions, and the underlying financial considerations, to technical uncertainty.

CONCLUSIONS

In this paper, a framework for coupled performance/financial aircraft design optimization is presented. The framework combines a real options valuation technique that explicitly addresses market uncertainty with a simulated annealing optimization algorithm. A simple example is considered to demonstrate the proof of concept. Some sensitivity studies have been performed to show the effect of changing aircraft weight on program

value. In the final paper, optimization results will be presented. In particular, financial-optimal and performance-optimal aircraft designs will be compared.

FIGURES

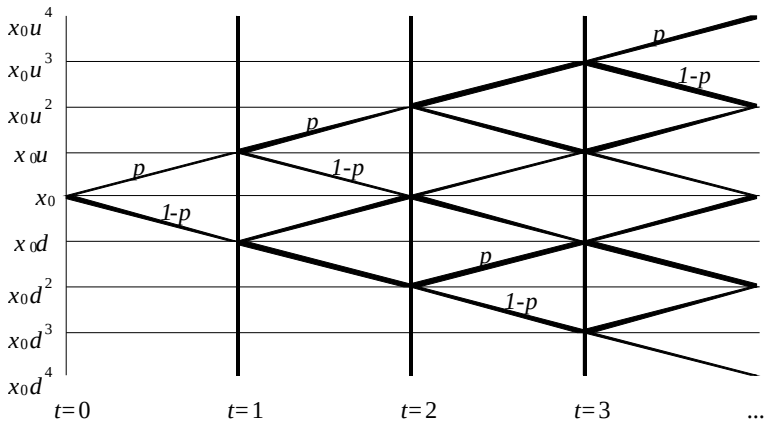

Figure 1. Binomial lattice of demand states and corresponding transition probabilities representing geometric Brownian motion model. (Markish)

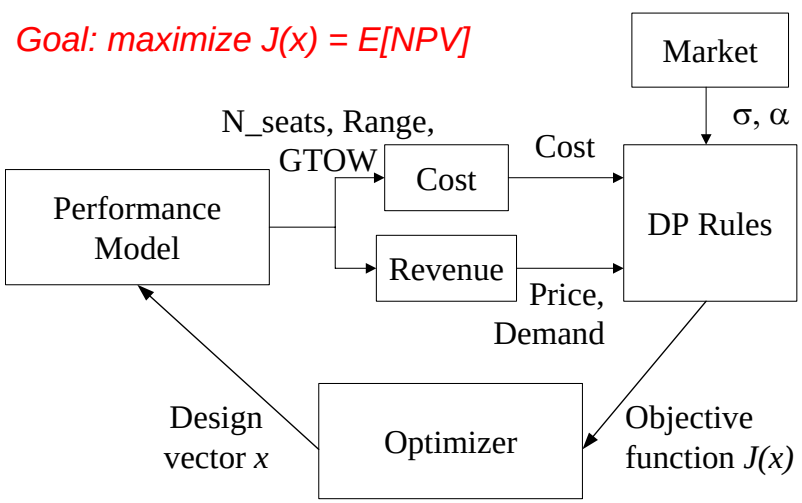

Figure 2. Optimization framework coupling performance and financial models for aircraft design.

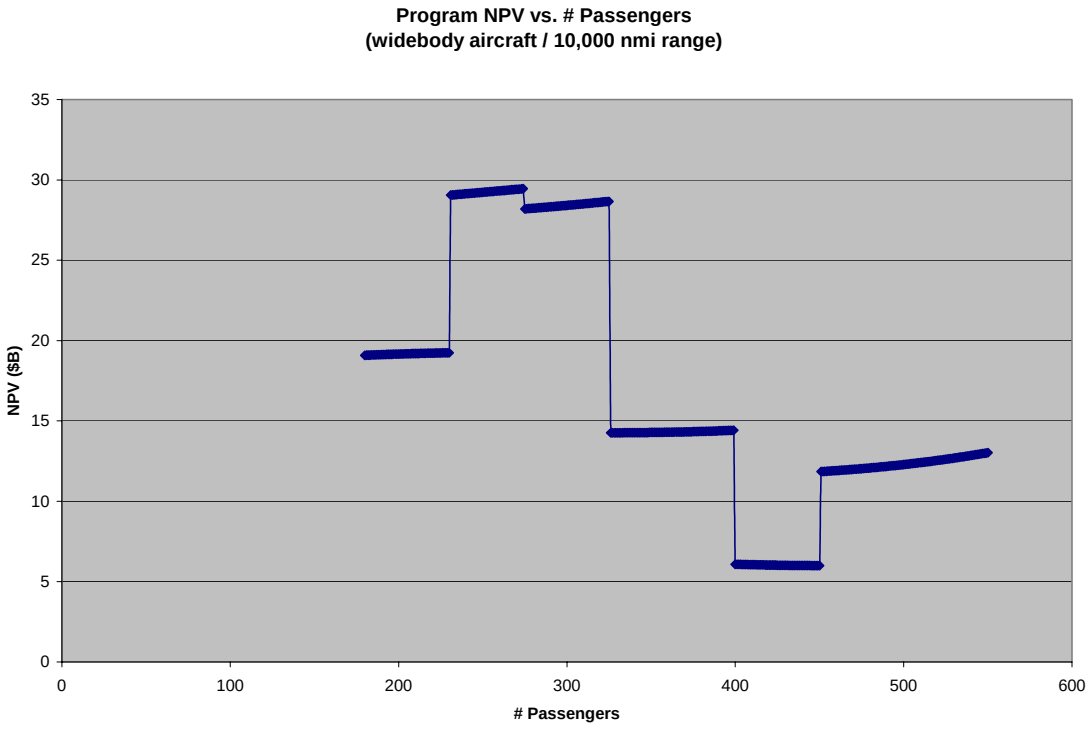

Figure 3. Expected NPV vs. number of passengers at an example range, demonstrating discontinuities in the design space due to shifting baseline demand as a function of number of passengers.

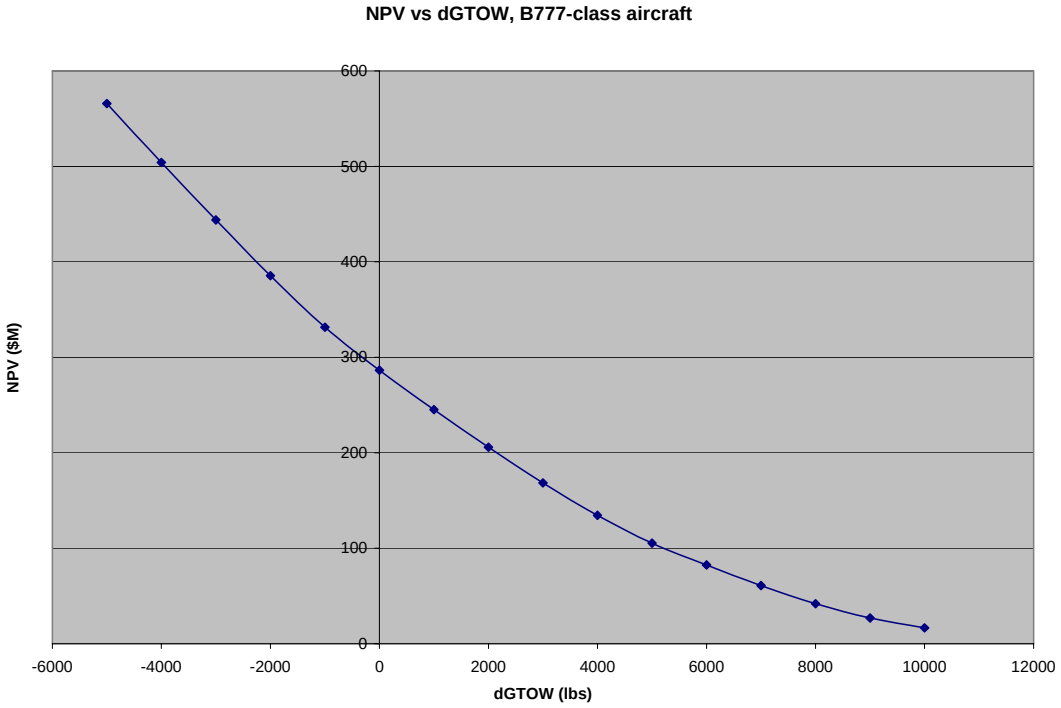

Figure 4. Effect of increasing GTOW on NPV using a real options approach allowing for managerial flexibility.

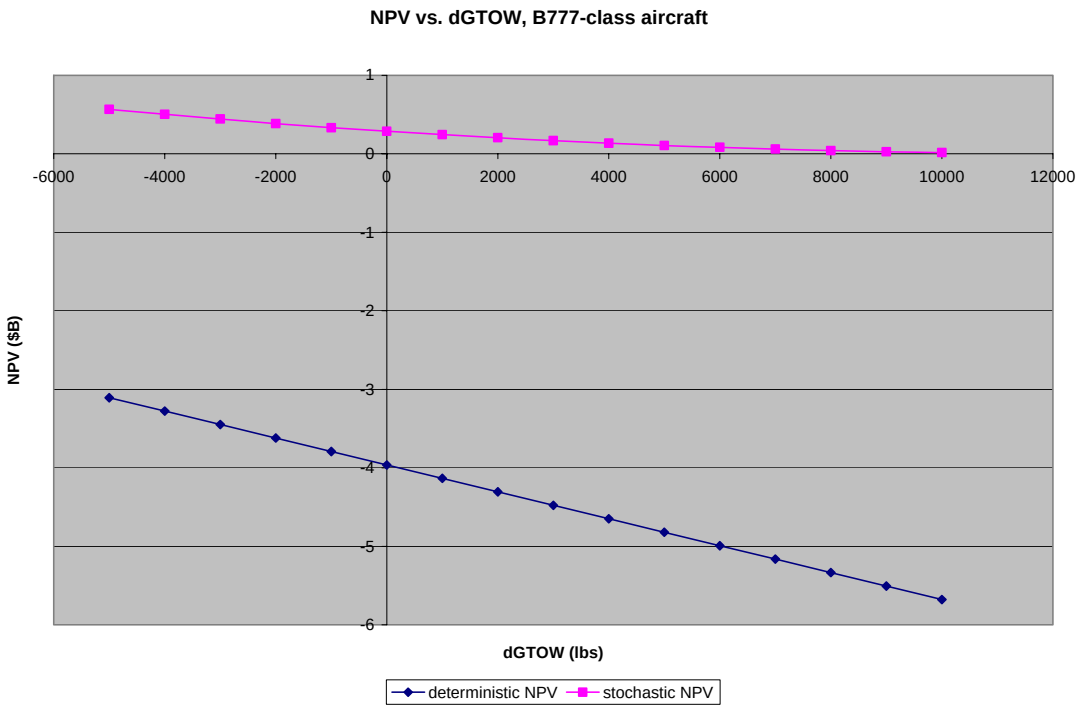

Figure 5. Effect of increasing GTOW on NPV with (stochastic, see Fig. 4) and without (deterministic) program flexibility.

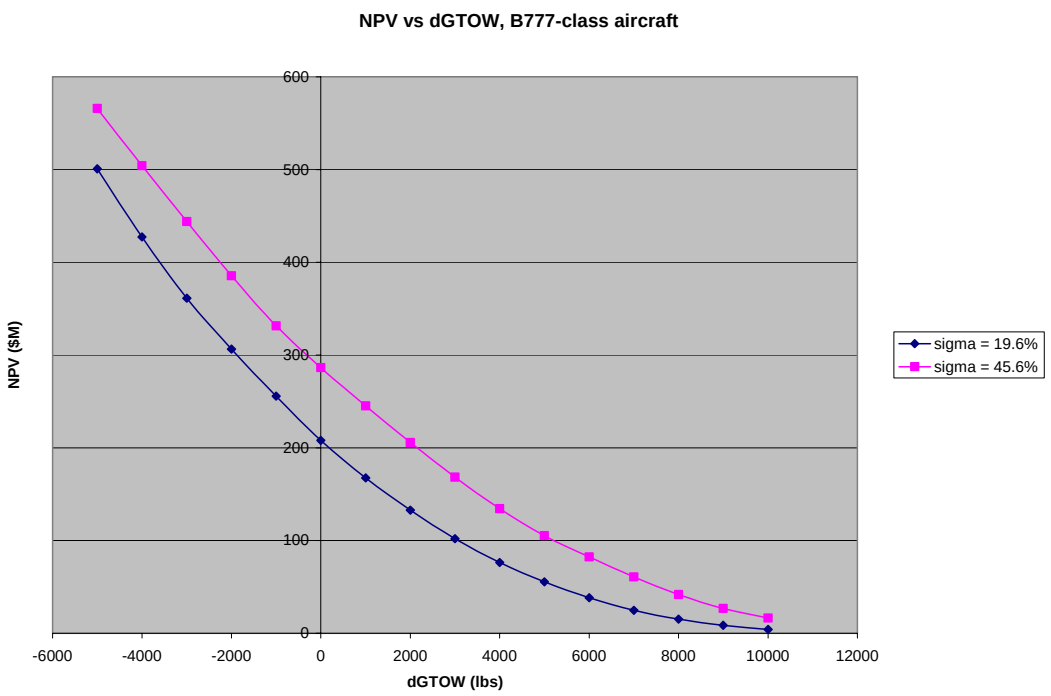

Figure 6. Comparison of NPV vs. increasing GTOW for baseline (avg. aircraft) and reduced (avg. widebody aggregate) volatilities.

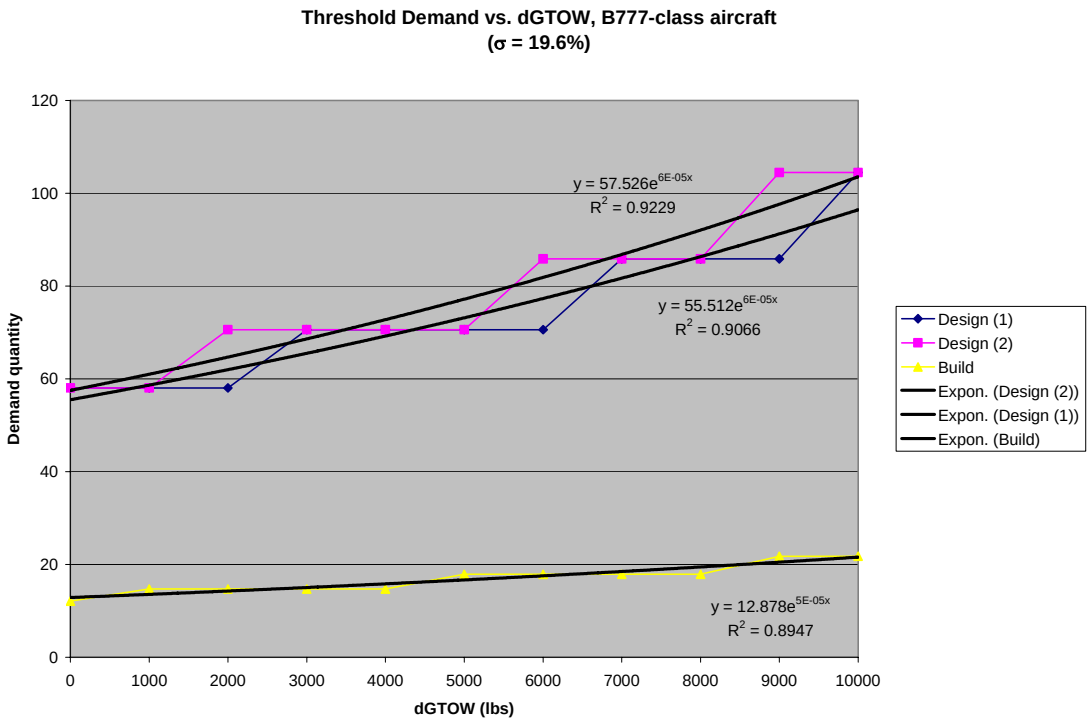

Figure 7. Initial threshold demand (t = 0) for design, tooling, and build phases of aircraft program vs. increasing GTOW.

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[1] Markish, J. *Valuation Techniques for Commercial Aircraft Program Design.* S.M. thesis. Massachusetts Institute of Technology, 2002.

[2] Liebeck, R.H. "Aircraft Sizing." Presentation to Aircraft Systems Engineering course at MIT, September 25, 2002.

[3] Dixit, A.K., and R.S. Pindyck. *Investment Under Uncertainty*. Princeton: Princeton University Press, 1994.

[4] Ingber, L. "Adaptive Simulated Annealing (ASA)." http://www.ingber.com/ASA-README.pdf, July 25, 2003.