Proceedings of ASME TURBO EXPO 2002
International Gas Turbine & Aeroengine Congress & Exhibition
June 3-6, 2002, Amsterdam, , The Netherlands

2002-GT-??

APPLICATION OF REDUCED-ORDER AERODYNAMIC MODELING TO THE ANALYSIS OF STRUCTURAL UNCERTAINTY IN BLADED DISKS

K. Willcox
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology
Cambridge, MA 02139
Email: kwillcox@mit.edu

J. Peraire
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology
Cambridge, MA 02139
Email: peraire@mit.edu

ABSTRACT

Blade-to-blade variations can significantly impact the operation of bladed disks. In this paper, a method is presented for assessing the effects of these variations using a high-fidelity aerodynamic analysis. Systematic model reduction is applied to a high-order computational fluid dynamics code using the proper orthogonal decomposition technique. This results in a low-order model suitable for time domain computations of mistuning effects. The model is shown to capture the dynamics of the aeroelastic system more accurately than with a traditional influence coefficient approach. Results are presented for a bladed disk with structural uncertainty, where the blade frequencies exhibit random variations about a nominal state. Finally, the concept of a robust design is explored, in which intentional variation is introduced to the system in an attempt to alleviate the ill-effects of random variations. The approach can also be extended to consider aerodynamic uncertainty, which may arise from geometric variations.

INTRODUCTION

In typical analyses of bladed disks, the problem is assumed to be tuned, that is all blades are assumed to have identical geometries, mass and stiffness characteristics. In reality, both the manufacturing process and engine wear create a situation where the blades differ slightly from one another. These blade-to-blade variations are known as mistuning. Even a small amount of mistuning can lead to a large asymmetric forced response (Dye and Henry, 1969). Mode shapes may become spatially localized, causing a single blade to experience deflections much larger than those predicted by a tuned analysis (Wei and Pierre, 1988a,b). Since forced response essentially determines high cycle fatigue or blade life, it is crucial to understand how mistuning affects the response.

It has also been shown that mistuning can increase the stability margin of a compressor (Kaza and Kielb, 1982), thus suggesting intentional mistuning as a form of passive control for flutter. The mistuning problem has been cast as a constrained optimization problem (Crawley and Hall, 1985; Shapiro, 1999) in which a deliberate mistuning pattern is chosen so as to maximize the stability margin of a blade row. From forced response considerations, the ideal state would be to have a perfectly tuned bladed disk. However there will always be some degree of *random mistuning* present due to limitations in the manufacturing process or due to engine wear. One can choose some *intentional mistuning* pattern so as to minimize the effect of these random variations. This idea of robust design has been discussed by several authors, including Castanier and Pierre (1998) and Shapiro (1998). The intentional mistuning is chosen so that the worst case forced response due to random variations about the intentionally mistuned design point is more acceptable than the worst case forced response due to random variations about the tuned design point.

Wei and Pierre (1988b) and Ottarsson and Pierre (1995) determined that moderately weak interblade coupling was required for the occurrence of significant forced response amplitude increases. Kruse and Pierre (1996a) consider two sources of interblade coupling: aerodynamic coupling and disk structural coupling. Aerodynamic coupling was found to be a signifi-

Copyright © 2002 by ASME

cant factor, increasing the vibratory stress levels by 70% over the tuned response. Kenyon and Rabe (1998) measured the response of an integrally bladed disk to inlet forcing, and compared the results to those predicted using a computational model. It was concluded that the response was strongly influenced by aerodynamic loading.

In all of these studies, the aerodynamic coupling was represented in the form of unsteady aerodynamic influence coefficients calculated at a single condition using a computational fluid dynamic (CFD) model. Kenyon and Rabe (1998) found that in their case the response was dominated by aerodynamic phenomena not effectively captured by the model, which led to an inaccurate prediction of the rotor response. It was concluded that more consideration must be given to the role of aerodynamic coupling in mistuned bladed disks. When mistuning is present, the discrete spatial modes present in the system do not decouple, and a much greater degree of aerodynamic coupling is observed. It is therefore not surprising that influence coefficients derived at a specific flow condition do not accurately capture the important dynamics.

Analysis of uncertainty in bladed disks is clearly an application that requires the use of more sophisticated aerodynamic models, although the need for computational efficiency is even more stringent due to the lack of cyclic symmetry in the problem. Any analysis (both structural and aerodynamic) must consider the full bladed disk, a prohibitively expensive proposition for conventional high-fidelity models such as computational fluid dynamic (CFD) methods or finite element methods (FEM). Model order reduction is a technique in which low-order models are derived from high-fidelity computational tools such as CFD or FEM. The derivation is done by systematically extracting the dynamics important for the specific problem of interest. This results in models that replicate high-fidelity results, but which have many fewer states.

Reduced-order structural models have been developed directly from finite element structural models (Ottarsson et al., 1994). These reduced-order models have been used to investigate the forced response of mistuned bladed disks and to examine the physical mechanisms associated with mistuning (Kruse and Pierre, 1996b). Reduced-order aerodynamic models have also been derived for analysis of bladed disks (Hall et al., 1994; Florea et al., 1996; Willcox et al., 2000; Epureanu et al., 2000, 2001; Dowell and Hall, 2001). These models allow the entire bladed disk to be considered with a reasonable number of states, typically on the order of ten states per blade passage for two-dimensional aeroelastic applications. The reduced-order aerodynamic models are also valid over a range of frequencies, thus capturing the important dynamics even when a significant amount of aerodynamic coupling exists.

Here, we consider a simple structural model coupled with a high-fidelity, reduced-order aerodynamic model. The aerodynamic model is derived from a CFD model of the two-dimensional Euler equations by using the proper orthogonal decomposition to systematically extract important dynamics. In this paper, the aeroelastic model is briefly described, then results are presented for analysis of a two-dimensional, transonic cascade in the presence of structural mistuning. The forced response and stability margin of this cascade are analyzed in the presence of random mistuning, and the results are compared to analysis using a traditional aerodynamic influence coefficient model. An example using intentional mistuning for robust design is also presented.

AEROELASTIC MODEL

Consider a general aeroelastic model of a bladed disk, which is written as

$$\frac{d\mathbf{w}}{dt} + f(\mathbf{w},\mathbf{x}) = 0. \tag{1}$$

Here $\mathbf{w}$ is a vector containing all the aerodynamic and structural states for the system and $\mathbf{x}$ represents the problem geometry. The aeroelastic model comprises two separate, yet coupled, components: the aerodynamic model and the structural dynamic model. Each of these can be derived using a multitude of methods with varying levels of fidelity. Here we present details for one particular set of models, although the methodology described applies in the general case.

For the aerodynamics we consider a CFD model of the two-dimensional Euler equations. Upon discretization of the governing equations, the aerodynamic model can be written as

$$\frac{d\mathbf{U}}{dt} + \mathbf{R}(\mathbf{U},\mathbf{U}_b,\mathbf{x}) = 0, \tag{2}$$

where $\mathbf{U}$ is the aerodynamic state vector containing the unknown flow quantities at each point in the computational domain. The vector $\mathbf{R}$ contains the nonlinear flux contributions at each node, which also depend on the problem geometry, $\mathbf{x}$, and the prescribed flow quantities at the domain boundaries, $\mathbf{U}_b$.

Similarly, we require a model that describes the structural dynamics of the bladed disk. We consider here a simple two degree of freedom model in which each blade can move rigidly in pitch, α, and plunge, h. The governing structural equations for each blade are therefore given by

$$M_i\ddot{\mathbf{q}}_i + C_i\dot{\mathbf{q}}_i + K_i\mathbf{q}_i = \mathbf{L}_i \tag{3}$$

where $\mathbf{q}_i$ contains the rigid displacements for blade i:

$$\mathbf{q}_i = [h_i \;\; \alpha_i]^T, \tag{4}$$

Copyright © 2002 by ASME

and M_i, C_i and K_i are the non-dimensional mass, damping and stiffness matrices for each blade i. These are given by the following well established relations:

$$M_i = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}_i \quad (5)$$

$$C_i = \begin{bmatrix} 2kM\zeta & 0 \\ 0 & 2kM\zeta\left(\frac{\omega_h}{\omega_\alpha}\right) r_\alpha \end{bmatrix}_i \quad (6)$$

$$K_i = \begin{bmatrix} k^2M^2 & 0 \\ 0 & k^2M^2\left(\frac{\omega_h}{\omega_\alpha}\right)^2 r_\alpha^2 \end{bmatrix}_i . \quad (7)$$

Here ω_h and ω_α are the uncoupled natural frequencies of the blade in plunge and pitch respectively, ζ is the structural damping coefficient, x_α is the non-dimensional distance of the center of gravity from the elastic axis, and r_α is the radius of gyration about the elastic axis. The reduced frequency is defined in terms of the plunge natural frequency, $k = \frac{\omega_h c}{V}$, and the load vector for each blade is

$$\mathbf{L}_i = \frac{2M^2}{\pi\mu}\begin{bmatrix} -C_l^i \\ C_m^i \end{bmatrix}, \quad (8)$$

where C_l^i is the lift coefficient for blade i and C_m^i is the moment coefficient about the aerodynamic center, which is located a distance a chord lengths in front of the elastic axis. M is the inlet Mach number and μ is the blade mass ratio. By using the identities $\frac{dh_j}{dt} = \dot{h}_j$ and $\frac{d\alpha_j}{dt} = \dot{\alpha}_j$, the structural system (3) can be written as a first order system as follows

$$\frac{d\mathbf{u}_i}{dt} = S_i\mathbf{u}_i + T_i\mathbf{y}_i, \quad (9)$$

where $\mathbf{u}_i = [\mathbf{q}_i \ \dot{\mathbf{q}}_i]^T$ contains the structural states for blade i, $\mathbf{y}_i$ contains the aerodynamic force and moment coefficients for blade i, and the matrices S_i and T_i follow from (3).

For this rigid structural model, the disk geometry $\mathbf{x}$ can be written in terms of the plunge displacement h_i and the pitch displacement α_i. For a disk with r blades, we write

$$\mathbf{x} = \mathbf{x}(\mathbf{q}), \quad (10)$$

where $\mathbf{q}^T = [\mathbf{q}_1^T \ \mathbf{q}_2^T \ldots \mathbf{q}_r^T]$. The rigid body displacements affect the aerodynamics via the boundary conditions, which are specified in the vector $\mathbf{U}_b$. We will also consider an external flow disturbance, which could, for example, be due to a neighboring blade row. Given blade motion $\mathbf{q}$ and external disturbance $\mathbf{d}$, $\mathbf{U}_b$ can be written

$$\mathbf{U}_b = \mathbf{U}_p(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{d}, \mathbf{x}), \quad (11)$$

where $\mathbf{U}_p$ is a vector containing the appropriate prescribed quantities.

Assuming all motions and disturbances are small, the governing aerodynamic equations (2) can be linearized as described in Willcox et al. (2000). The resulting aerodynamic system is written as

$$\frac{d\mathbf{U}'}{dt} = A\mathbf{U}' + B\mathbf{u} + E\mathbf{d} \quad (12)$$

where $\mathbf{U}'$ contains the perturbation flow quantities and the matrices A, B and E contain the appropriate linearization terms, which are all evaluated at steady-state conditions. Equation (12) shows how the structural state vector $\mathbf{u}$ enters into the aerodynamics. Examining (9) we notice that the aerodynamics drive the structural dynamics through the force and moment coefficients contained in the vector $\mathbf{y}$. These forces and moments can be expressed in terms of the perturbation flow quantities as

$$\mathbf{y} = C\mathbf{U}' \quad (13)$$

where C is a matrix containing the linearized force calculation.

We can now combine equations (9), (12) and (13) to determine the evolution of the combined linearized aeroelastic state $\mathbf{w} = [\mathbf{U}' \ \mathbf{u}]^T$:

$$\dot{\mathbf{w}} = \begin{bmatrix} A & B \\ TC & S \end{bmatrix}\mathbf{w} + \begin{bmatrix} E \\ 0 \end{bmatrix}\mathbf{d}. \quad (14)$$

Low-Order Aerodynamic Models

The aeroelastic state vector $\mathbf{w}$ in (14) is very large - for a two-dimensional Euler problem there are tens of thousands of aerodynamic states per blade passage. In order to evaluate the effect of mistuning on the system, we require both a stability analysis and forced response for the entire bladed disk. Such computations are not practicable with such a high-order model, hence there is a need to derive low-order models. Here we describe two methods, both of which originate from a high-order CFD method. The first approach is to use the CFD model to evaluate the aerodynamic response at a particular flow condition, resulting in a set of influence coefficients. The second method uses systematic model order reduction to extract relevant dynamics from the CFD model, resulting in a low-order state-space system.

Aerodynamic Influence Coefficients. In the past, the need for low-order models has been resolved through the use of simple assumed-frequency models for the aerodynamic portion. The high-order CFD model (12) is used to calculate the disk response to a particular prescribed set of inputs. This calculation

Copyright © 2002 by ASME

results in a set of influence coefficients, which are coupled to the structural model and are assumed to represent the response for all flows. For blade motion, these influence coefficients represent the magnitude of the forces generated on each blade due to an imposed unit sinusoidal motion on one blade and all other blades fixed. For external forcing, they represent the forces generated on each blade due to a unit sinusoidal disturbance in the appropriate flow quantity.

Since this approach assumes fixed dynamics, it will be inaccurate, especially in the mistuning context where a high degree of aerodynamic coupling may be present. Typically, the eigenvalues of a mistuned system are expected to exhibit a sufficiently high degree of scatter so that assumed-frequency models do not provide accurate results. This will be demonstrated in the examples presented in this paper.

Model Order Reduction. Systematic model reduction provides a way to obtain a low-order aerodynamic model while retaining the high-fidelity dynamics over a range of forcing inputs. Many techniques have been studied and applied to a variety of problems. Here we apply the proper orthogonal decomposition (POD) to the CFD model (12). The details of this approach can be found in Willcox et al. (2000) and Willcox (2000). Once the reduction has been performed, we obtain a state-space system identical in form to (12) and (13) but with many fewer states. We write

$$\frac{d\mathbf{v}}{dt} = A\mathbf{v} + B\mathbf{u} + E\mathbf{d} \tag{15}$$

$$\mathbf{y} = C\mathbf{v} \tag{16}$$

where $\mathbf{v}$ is the reduced-space aerodynamic state vector and A, B, C and E are matrices describing the reduced-order state-space system.

Upon coupling with the structural model (9), we therefore obtain a low-order aeroelastic model of the form (14) that can be used extensively for time domain analysis. We also note that one could start with a high-order, high-fidelity, structural model (for example a finite element analysis) and perform a similar reduction procedure to obtain a reduced-order structural model as in Ottarsson et al. (1994) and Kruse and Pierre (1996b).

MISTUNED TRANSONIC CASCADE

The test case considered is the DFVLR transonic cascade, an experimental cascade discussed in Youngren (1991). The examples selected for analysis have a steady-state flow with an inlet Mach number of 0.82 at a relative flow angle of 58.5°. The cascade is analyzed with twenty blades, each of which moves in unsteady plunging motion with a tuned natural reduced frequency chosen to be $k = 0.122$. The reduced-order model obtained via the POD contains four aerodynamic states per blade passage, for a total of 120 states in the full cascade aeroelastic model. An axial velocity defect was also admitted at the cascade inlet with twenty possible disturbance spatial frequencies included in the model.

One difficulty when deriving a reduced-order model is determining the number of states to retain. Typically, the procedure is to derive a reduced-order model and then compare simulation results with those obtained from the high-order CFD code. The reduced-order model used here was validated by choosing several representative test cases which spanned both frequencies and interblade phase angles of interest. These validation results can be found in Willcox (2000), and show that the dynamics of interest can be captured with four reduced-order states per interblade phase angle, for a total of 80 aerodynamic states in the reduced-order model. A relatively small frequency range is considered here, hence the number of POD modes required for accuracy is low. In other aeroelastic applications, the number of states is typically on the order of ten per interblade phase angle (Willcox, 2000; Epureanu et al., 2000, 2001). It is also interesting to note that the total number of aerodynamic states could be further reduced by allowing the POD modes to span interblade phase angles in addition to frequencies, as in Epureanu et al. (2001).

To demonstrate the effects of mistuning on the cascade response, structural parameters are chosen so as to obtain a very lightly damped system. The case chosen has a blade mass ratio of $\mu = 100$ and a structural damping of $\zeta = -0.0186$. Note that a small negative value of structural damping has been chosen. Clearly this is not physical, however it is used to establish a system which is very lightly damped, and which therefore will exhibit a large sensitivity to mistuning. Such a mode may actually exist in many physical systems, so it is important to determine the possible implications in a mistuning context and to understand their sources.

It is relatively straightforward to include the effects of structural mistuning in the aeroelastic model. For example, if we consider frequency mistuning, then the frequency for blade i is given by

$$\omega_i = \omega_0(1 + z_i), \tag{17}$$

where ω_0 is the nominal or tuned frequency and z_i is the mistuning for blade i. The aeroelastic system can therefore be written

$$\dot{\mathbf{w}}_r = \begin{bmatrix} A & B \\ TC & S(\mathbf{z}) \end{bmatrix} \mathbf{w}_r + \begin{bmatrix} E \\ 0 \end{bmatrix} \mathbf{d}, \tag{18}$$

where $\mathbf{w}_r = [\mathbf{v} \quad \mathbf{u}]^T$. The effect of the mistuning on stability can be determined by evaluating the eigenvalues of (18) and comparing them to the tuned system. Since (18) contains only 120

Copyright © 2002 by ASME

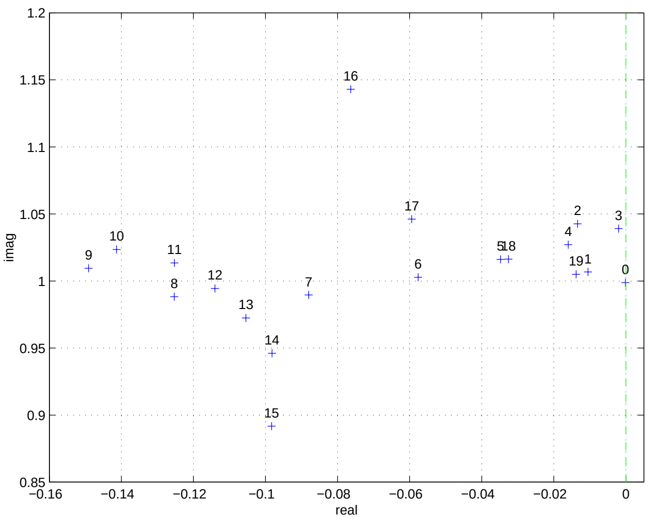

Figure 1. Tuned structural eigenvalues for reduced-order model. $k = 0.122$, $\mu = 100$, $\zeta = -0.0186$. Eigenvalues are numbered by their nodal diameter.

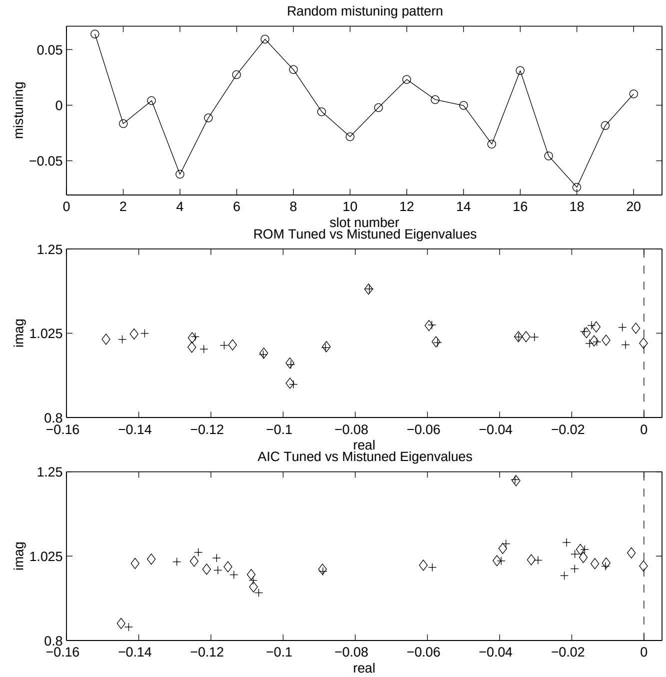

Figure 2. Random mistuning of DFVLR cascade. Top: random mistuning pattern. Middle: reduced-order model tuned eigenvalues (diamonds) and mistuned eigenvalues (plus signs). Bottom: influence coefficient model tuned eigenvalues (diamonds) and mistuned eigenvalues (plus signs).

states, time domain simulations can be easily performed to determine forced response.

The tuned eigenvalues for this system that correspond to the structural states are plotted in Figure 1. The numbers on the plot are the number of nodal diameters, which indicate the spatial frequency associated with each eigenmode. For ℓ nodal diameters, the corresponding interblade phase angle is $\sigma_\ell = 2\pi\ell/20$. The frequencies fall close to the damped natural frequency of $\sqrt{1-\zeta^2} = 1.00$. The $\ell = 0$ structural mode is barely stable, and the $\ell = 3$ mode is very lightly damped.

Random Mistuning

We now apply a random mistuning to the structural frequencies of the blades, generated by a normal distribution with a zero average and a 4% variance. The random mistuning pattern considered is shown in Figure 2 along with the mistuned and tuned structural eigenvalues for the reduced-order model. It can be seen that the lightly damped mistuned eigenvalues are to the left of the tuned ones, and so this mistuning pattern stabilizes the system, which is true for most mistuning (Bendiksen, 1983). As noted in Crawley (1988), the centroid of the structural eigenvalues cannot be altered by a zero-average mistuning. In Figure 2 we see that while the lightly damped modes are stabilized, the highly damped eigenvalues shift to the right in order to maintain the position of the centroid. The degree of scattering of the eigenvalues about the centroid is dependent on the amount of coupling between the aerodynamics and the structure. Figure 2 also shows that the mistuning reduces the influence of the aerodynamic coupling and moves the eigenvalues towards the centroid, as also discussed in Crawley (1988).

Comparison with Influence Coefficient Model

The results presented here will show that when mistuning is present, an aerodynamic influence coefficient model cannot accurately capture the system behavior. An identical random mistuning pattern was applied to the influence coefficient model with the same structural parameters described above, and the eigenvalues and forced response of the system were evaluated. Figure 2 also shows the tuned and mistuned eigenvalues for the influence coefficient model. We notice that the mistuning causes the eigenvalues of the influence coefficient model to move much more than those of the reduced-order model. In Figure 3 the same eigenvalues are plotted but now the tuned and mistuned eigenvalues for each model are compared. In the mistuned case, there is a much greater error in the influence coefficient eigenvalues.

It has been shown that for tuned systems, influence coefficient models do not predict the correct eigenvalue when the frequency shifts significantly from the natural frequency or when a significant amount of aerodynamic damping is present (Willcox et al., 2000). The influence coefficient model is therefore expected to provide a much worse estimate of the eigenvalues

Copyright © 2002 by ASME

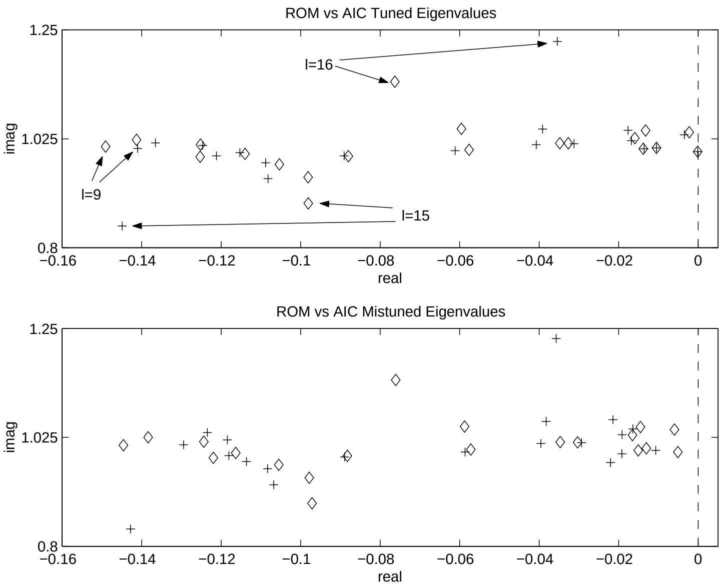

Figure 3. Random mistuning of DFVLR transonic cascade. Top: tuned eigenvalues - reduced-order model (diamonds) and influence coefficient model (plus signs). Bottom: mistuned eigenvalues - reduced-order model (diamonds) and influence coefficient model (plus signs).

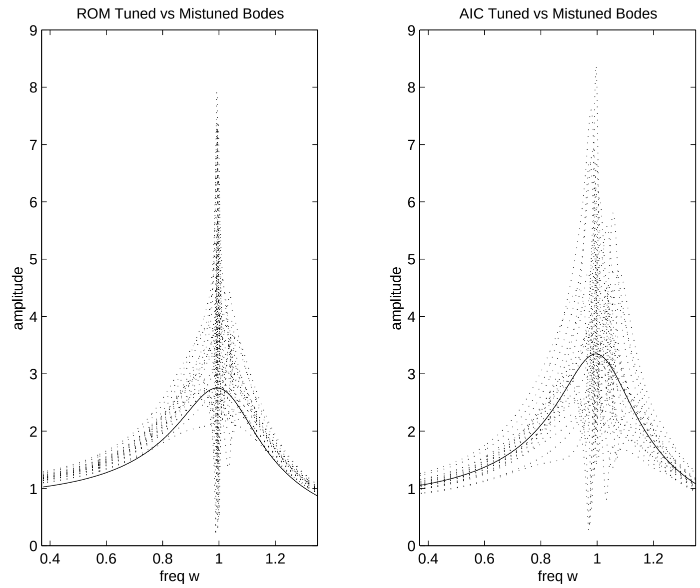

Figure 4. Random mistuning of DFVLR transonic cascade. Forced response to inlet disturbance in the $\ell = 9$ mode for reduced-order model (left) and influence coefficient model (right). Solid line denotes the tuned response, dotted lines are the mistuned response.

when mistuning is present, since, even though the influence of the aerodynamic coupling is reduced, the interblade phase angles no longer decouple. In the tuned system, only the aerodynamics for one interblade phase angle contribute to the placement of each eigenvalue. If a particular mode happens to have an eigenvalue that falls close to the natural frequency with a small amount of damping (corresponding to the case of low aerodynamic coupling), then the influence coefficient model does an excellent job of predicting the eigenvalue position. This can be seen in the top plot of Figure 3 where the eigenvalues satisfying the above requirements agree closely with the reduced-order model. However in the mistuned system, the interblade phase angles do not decouple and all dynamics are relevant in computing each eigenvalue. Therefore, if *any* modes exist whose influence coefficients are not representative for the tuned system, the eigenvalues for the mistuned system will be inaccurate and the stability margin of the system will be mispredicted. This can be seen in the lower plot of Figure 3 where the difference between the reduced-order model and the influence coefficient model eigenvalues is significant for *all* modes.

Forced Response

The cascade is forced with an axial velocity inlet disturbance in the ninth nodal diameter mode ($\ell = 9$), which corresponds to the most highly damped tuned eigenvalue in Figure 1 (this mode is marked in Figure 3). The forced response predicted by each model is shown in Figure 4 for the tuned case (solid lines) and the mistuned case (dotted lines). When the system is tuned, forcing in the ninth spatial mode excites a response in only that mode, and all blades have the same response amplitude, thus the tuned forced response is a single highly damped smooth line. When the system is mistuned, the spatial modes no longer decouple, and forcing in the ninth spatial mode excites all of the structural eigenvalues, including the very lightly damped $\ell = 0$ and $\ell = 3$ modes. Each blade also now exhibits a different response amplitude. Because the lightly-damped modes are now present in the response, we see sharp peaks in the mistuned Bode plot at the frequencies corresponding to the relevant eigenvalues. Here, several blades have a large peak near $\omega = 1$, which corresponds to the very lightly damped $\ell = 0$ mode. We also see a smaller peak for one blade near $\omega = 1.05$, which corresponds to the $\ell = 3$ mode.

The influence coefficient model in fact does a very good job of predicting the response, even when mistuning is present. The peak tuned response amplitude is slightly higher than that predicted by the reduced-order model, since the tuned influence coefficient eigenvalue is less highly damped. The mistuned response is computed surprisingly accurately by the influence coefficient model, despite the errors in the mistuned eigenvalue predictions. Inspection of the mistuned eigenvalues in Figure 3 shows that the frequencies of the lightly damped $\ell = 0$ and $\ell = 3$ modes are computed accurately, therefore the peaks of the forced response in Figure 4 occur at the correct frequency. The damping

Copyright © 2002 by ASME

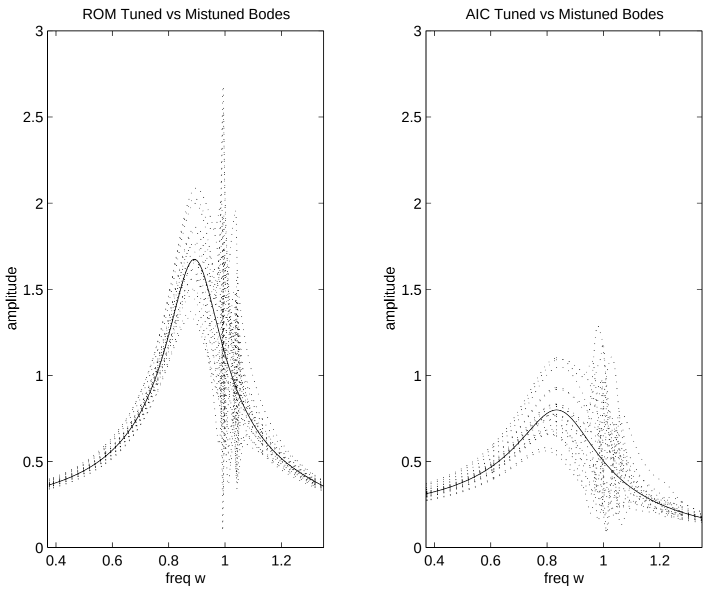

Figure 5. Random mistuning of DFVLR transonic cascade. Forced response to inlet disturbance in the $\ell = 15$ mode for reduced-order model (left) and influence coefficient model (right). Solid line denotes the tuned response, dotted lines are the mistuned response.

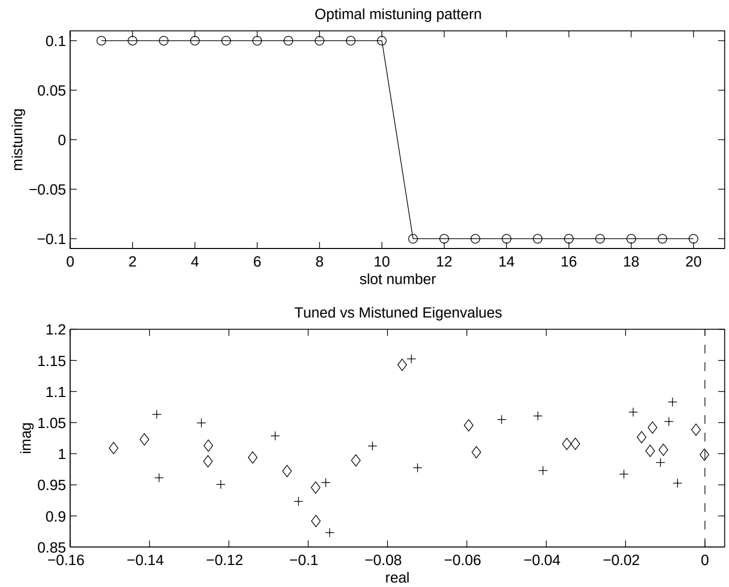

Figure 6. Optimal mistuning of DFVLR cascade. Top: optimal mistuning pattern. Bottom: reduced-order model tuned eigenvalues (diamonds), mistuned eigenvalues (plus signs). Result taken from Shapiro (1999).

of these two modes is predicted to be higher than it should, however this may be compensated by the fact that the tuned $\ell = 9$ damping is underpredicted, thus resulting in almost the correct forced response amplitude.

The two modes whose frequencies do move significantly from the natural frequency are $\ell = 15$ and $\ell = 16$. As was shown in Willcox et al. (2000), when a frequency shift occurs, the influence coefficients do not model the dynamics accurately. This is demonstrated by the difference in position for the $\ell = 15$ and $\ell = 16$ eigenvalues between the reduced-order model and influence coefficient model in both plots in Figure 3. When the forced response is calculated for one of these modes, the influence coefficient model no longer predicts the amplitude accurately. Figure 5 shows the forced response calculated for the two models for inlet disturbance forcing in the fifteenth spatial mode. We notice first that the tuned forced response predictions differ. This is because the damping of the tuned $\ell = 15$ eigenvalue is incorrectly predicted by the influence coefficient calculation. Figure 3 shows that the damping of the eigenvalue is significantly overpredicted by the influence coefficient model, which is consistent with the lower forced response amplitude. When mistuning is introduced into the system, the influence coefficient model does not capture the true amplitudes of the peaks associated with lightly damped modes.

Intentional Mistuning

Although the random mistuning appears to be beneficial in that it stabilizes the system by moving the lightly damped eigenvalues to the left, it creates a situation where the forced response amplitude may rise to unacceptable levels, and also introduces high loading on some individual blades. This might create a problem in practice if a disturbance is known to exist in a particular spatial mode whose eigenvalue is highly damped. A tuned analysis would predict a low forced response amplitude, while in reality small blade to blade variations exist, and the actual response may contain components of the lightly damped modes as demonstrated by Figure 4.

The idea behind robust design is to find an intentionally mistuned design point for the blades where the forced response due to random mistuning will be more acceptable than that shown in Figure 4. The intentional mistuning is chosen so as to optimize the following objective :

$$\text{Maximize } \Delta\zeta(\mathbf{z}) \text{ subject to } ||\mathbf{z}||_\infty \leq 0.1 \text{ and } \sum z_i = 0. \quad (19)$$

This means that we are finding the zero-average mistuning that provides the maximum increase in stability - it drives the least stable eigenvalue pair as far to the left as possible, subject to a constraint on the size of the mistuning. The optimal solution was determined in Shapiro (1999) and is shown in Figure 6. The corresponding eigenvalue plot shows that the least stable $\ell = 0$ and $\ell = 3$ eigenvalues have been pushed a significant amount to the left.

We now consider the same random mistuning applied about this intentionally mistuned point. The optimal plus random mistuning pattern is shown in Figure 7 along with the corresponding eigenvalues. Once again, we force in the ninth spatial mode

Copyright © 2002 by ASME

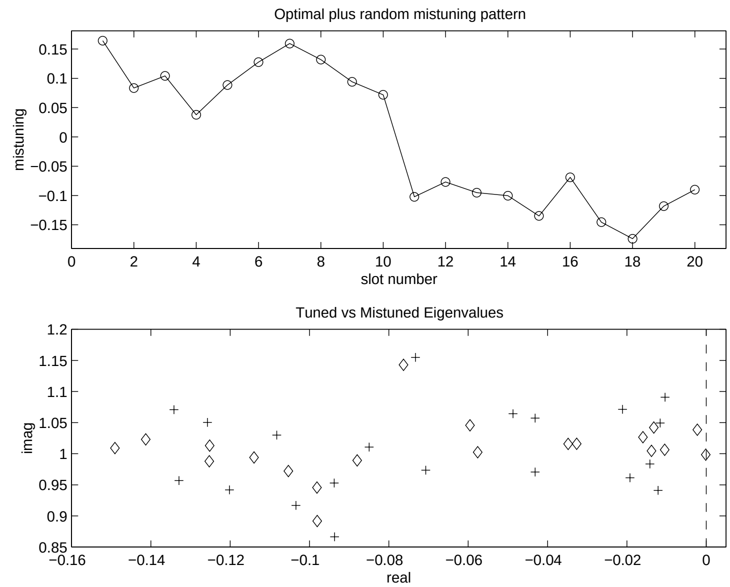

Figure 7. Optimal plus random mistuning of DFVLR cascade. Top: optimal plus random mistuning pattern. Bottom: reduced-order model tuned eigenvalues (diamonds), mistuned eigenvalues (plus signs).

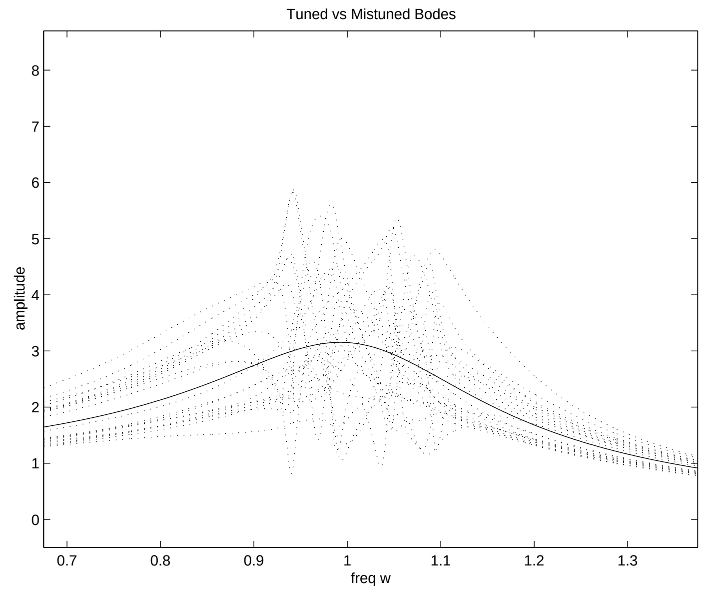

Figure 8. Optimal plus random mistuning of DFVLR cascade. Forced response of tuned system (solid line) and mistuned system (dotted lines) to an inlet disturbance in the ninth spatial mode.

and compute the response of the tuned and mistuned systems. The Bode plots shown in Figure 8 demonstrate that although the forced response of the mistuned system (dotted lines) is higher than that of the tuned system (solid line), the worst-case amplitude has been significantly reduced compared with that shown in Figure 4 for the same random mistuning pattern. The sensitivity of the forced response to random mistuning has been significantly decreased by the introduction of intentional mistuning.

CONCLUSIONS

In analysis of mistuned bladed disks, focus has been on developing high-fidelity structural models. Traditionally, the aerodynamics have been represented by an influence coefficient model that assumes fixed dynamics. However, when mistuning is present, symmetry is destroyed and the spatial frequencies no longer decouple. This leads to greater inaccuracy in the predicted aeroelastic coupling for the influence coefficient models and they therefore cannot predict mistuned behavior accurately. This inaccuracy is realized by error in both stability margin and forced response predictions. The transonic cascade example presented here highlights this point.

Until now, the alternative for better aerodynamic models has been high-order CFD methods. However, the size and computational expense of these methods diminishes their utility in a mistuning context where the entire blade row must be considered. Systematic model reduction provides a means to obtain low-order, high-fidelity models that are valid over a range of forcing conditions. Results presented here show that these reduced-order models are more appropriate for analysis of mistuned bladed disks than the traditional influence coefficient approach. Moreover, these models can been extended to include the effects of geometric variations between blades. Although past efforts have largely focused on structural mistuning, in practice the problem of aerodynamic mistuning is of significant importance. In order to capture the effects of variation in blade geometric parameters (such as shape and twist angle), the aerodynamic model must be considerably more sophisticated than a simple influence coefficient approach. Current efforts are being focused on extending the application of the reduced-order aerodynamic models described here to consider aerodynamic mistuning.

ACKNOWLEDGMENT

The authors would like to thank Prof. Ben Shapiro for the use of his software package MAST, which was used to generate the mistuning results presented here.

REFERENCES

Bendiksen O., 1983. “Flutter of Mistuned Turbomachinery Rotors.” ASME 83-GT-153.

Castanier M. and Pierre C., 1998. “Investigation of the Combined Effects of Intentional and Random Mistuning on the Forced Response of Bladed Disks.” AIAA Paper 98-3720.

Crawley E., 1988. “Aeroelastic Formulation for Tuned and Mistuned Rotors.” AGARD Manual on Aeroelasticity in Axial-Flow Turbomachines, Volume 2.

Copyright © 2002 by ASME

Crawley E. and Hall K., 1985. “Optimization and Mechanisms of Mistuning in Cascades.” *Transactions of the ASME*, vol. 107 pp. 418–426.

Dowell E. and Hall K., 2001. “Modeling of fluid-structure interaction.” *Annual Review of Fluid Mechanics*, vol. 33 pp. 445–90.

Dye R. and Henry T., 1969. “Vibration Amplitudes of Compressor Blades Resulting from Scatter in Blade Natural Frequencies.” *ASME Journal of Engineering for Power*, vol. 91 pp. 182–188.

Epureanu B., Dowell E., and Hall K., 2000. “Reduced-Order Models of Unsteady Transonic Viscous Flows in Turbomachinery.” *Journal of Fluids and Structures*, vol. 14 pp. 1215–34.

Epureanu B., Dowell E., and Hall K., 2001. “A Parametric Analysis of Reduced Order Models of Potential Flows in Turbomachinery using Proper Orthogonal Decomposition.” ASME paper 2001-GT-0434.

Florea R., Hall K., and Cizmas P., 1996. “Reduced Order Modelling of Unsteady Viscous Flow in a Compressor Cascade.” AIAA Paper 96-2572.

Hall K., Florea R., and Lanzkron P., 1994. “A Reduced Order Model of Unsteady Flows in Turbomachinery.” ASME Paper 94-GT-291.

Kaza K. and Kielb R., 1982. “Flutter and Response of a Mistuned Cascade in Incompressible Flow.” *AIAA Journal*, vol. 20, 8 pp. 1120–1127.

Kenyon J. and Rabe D., 1998. “Aerodynamic Effects on Blade Vibratory Stress Vibrations.” AIAA Paper 98-3744, Lake Buena Vista, Florida.

Kruse M. and Pierre C., 1996a. “Dynamic Response of an Industrial Turbomachinery Rotor.” Proceedings of the 32nd AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit.

Kruse M. and Pierre C., 1996b. “Forced Response of Mistuned Bladed Disks using Reduced-Order Modeling.” AIAA Paper 96-1545-CP.

Ottarsson G., Castanier M., and Pierre C., 1994. “A Reduced-Order Modeling Technique for Mistuned Bladed Disks.” Proceedings of the 35th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference.

Ottarsson G. and Pierre C., 1995. “On the Effects of Interblade Coupling on the Statistics of Maximum Forced Response Amplitudes in Mistuned Bladed Disks.” Proceedings of the 36th AIAA/ASME Structures, Structural Dynamics, and Materials Conference.

Shapiro B., 1998. “Symmetry Approach to Extension of Flutter Boundaries via Mistuning.” *Journal of Propulsion and Power*, vol. 14, 3 pp. 354–366.

Shapiro B., 1999. *Passive Control of Flutter and Forced Response in Bladed Disks via Mistuning*. Ph.D. thesis, California Institute of Technology.

Wei S. and Pierre C., 1988a. “Localization Phenomena in Mistuned Assemblies with Cyclic Symmetry, Part I: Free Vibrations.” *ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design*, vol. 110, 4 pp. 429–438.

Wei S. and Pierre C., 1988b. “Localization Phenomena in Mistuned Assemblies with Cyclic Symmetry, Part II: Forced Vibrations.” *ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design*, vol. 110, 4 pp. 439–449.

Willcox K., 2000. *Reduced-Order Aerodynamic Models for Aeroelastic Control of Turbomachines*. Ph.D. thesis, Dept. of Aeronautics and Astronautics, MIT.

Willcox K., Peraire J., and Paduano J., 2000. “Application of Model Order Reduction to Compressor Aeroelastic Models.” ASME International Gas Turbine and Aeroengine Congress, paper number 2000-GT-0377, Munich, Germany.

Youngren H., 1991. *Analysis and Design of Transonic Cascades with Splitter Vanes*. Master’s thesis, Dept. of Aeronautics and Astronautics, MIT.

Copyright © 2002 by ASME