SANS Contributions to Case CI1

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Solution Adaptive Numerical Simulator (SANS):

- Stabilized-CG, DG, HDG, EDG discretizations
  - DG-BR2 used here
- Automatic Differentiation
- Linear Solvers
  - GMRES
  - UMFPACK, MKL-PARDISO, and PETSc interfaces (MKL-PARDISO used here)
- Non-linear Solver
  - Newton-Raphson with line search
  - PTC and P-Sequencing used here
- Output-based mesh adaption using MOESS

  minimize estimated error in output functional

  subject to DOF count $\leq$ set cost

Regularization

Euler equations augmented with artificial viscosity,

$$\nabla \cdot \mathbf{F}(\mathbf{u}) - \nabla \cdot \left(\epsilon_s \tilde{\mathcal{A}} \nabla \mathbf{u}\right) = 0, \tag{1}$$

ϵ_s is the artificial viscosity
$\tilde{\mathcal{A}}$ applies viscosity to total enthalpy, not total energy

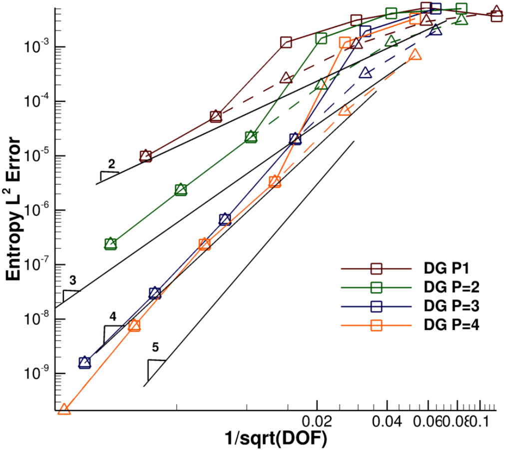
Artificial viscosity based on Barter (2010) artificial viscosity PDE:

$$\nabla \cdot \left(C_2 H^2 \nabla s\right) + \left[\tilde{S}_k(\mathbf{u}) - s\right] = 0, \tag{2}$$

- PDE for a sensor field (s – Peclet number for ϵ_s)
- $\tilde{S}_\kappa$ – Inter-element pressure jumps detect shocks
- H – Generalized length scale

Verification: Gaussian Bump

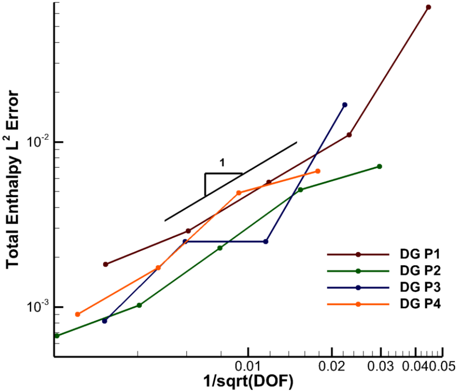
L^2 norm of entropy in the domain with and without artificial viscosity
Triangle grid with Q=4

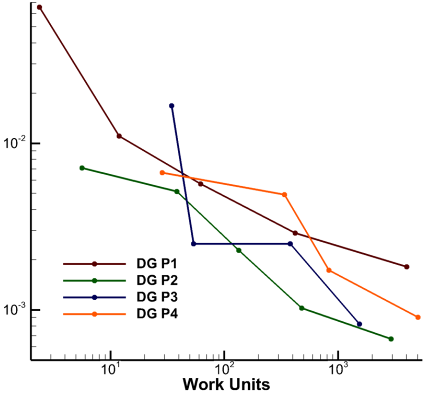


Expected $p+1$ convergence in asymptotic regime

HOW5 CI1 - Provided Grids

L^2 norm of stagnation enthalpy in the domain



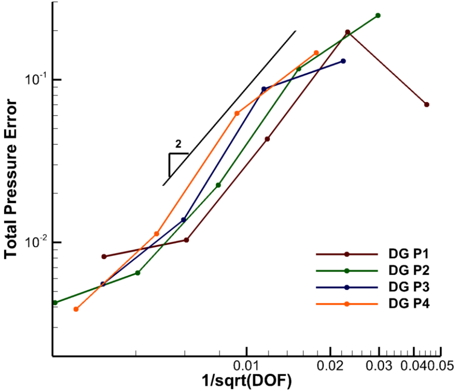


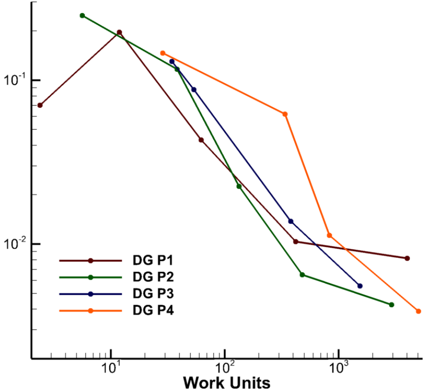
Approx. 1^{st} order convergence.

HOW5 CI1 - Provided Grids

Gaussian weighted ($g(y)$) total pressure error:

$$\bar{P}_t = \int_{\Gamma_w} g(y)\,(P_t(y) - P_{02})\,dy$$



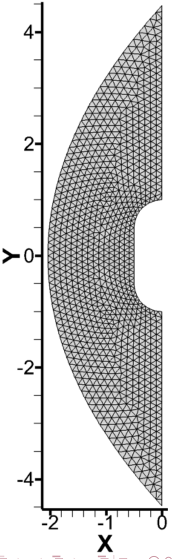


Approx. 2^{nd} order convergence

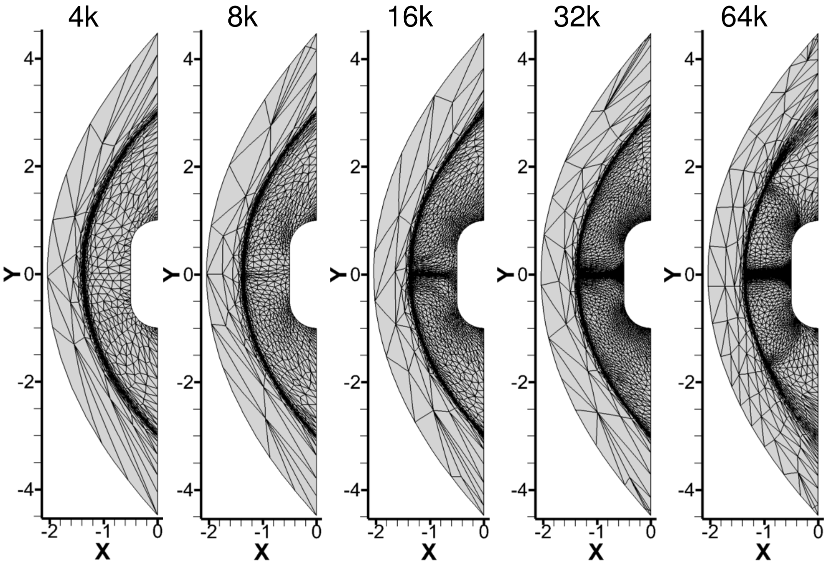
HOW5 CI1 - Adaptation Setup

- All grids are curved with Q=3
  - Boeing mesh generator EPIC
- Fixed polynomial degree:
  - P=1, P=2, P=3
- Target costs:
  - 4k, 8k, 16k, 32k, 64k
- Output functionals:
  - Enthalpy error: $\int_\Omega (H - H_\infty)^2$
  - Stagnation pressure error: $\bar{P}_t$
- 30 adaptation iterations per configuration
- Total: 900 grids
  - Adjoint requires machine zero residuals

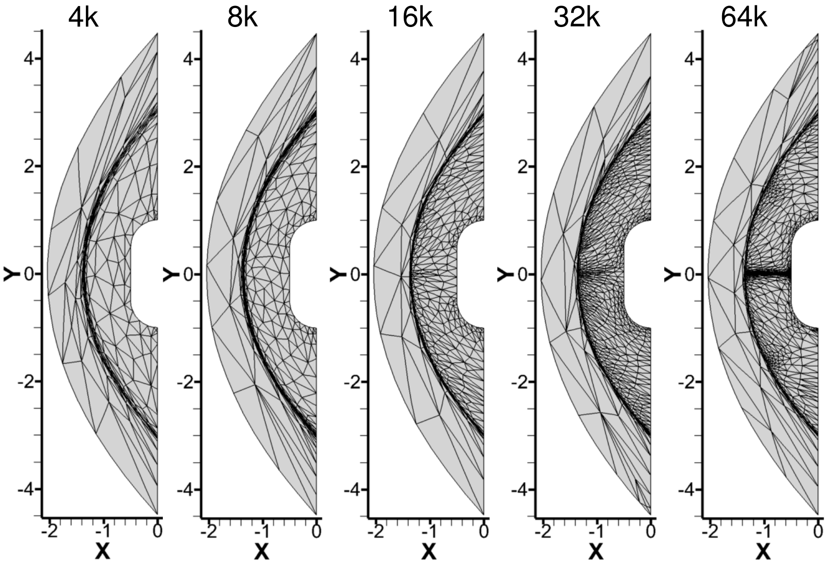
Initial Grid



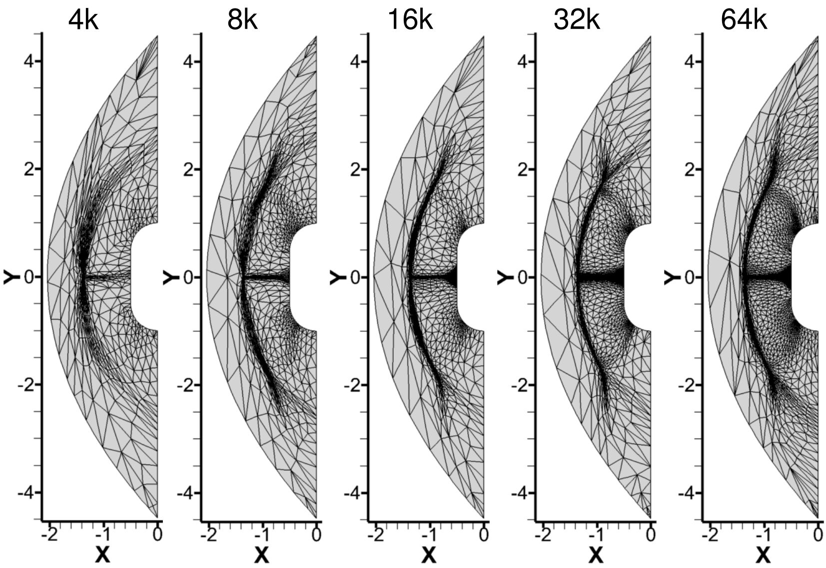
HOW5 CI1 - Adapted Meshes (Enthalpy Error, P=1)



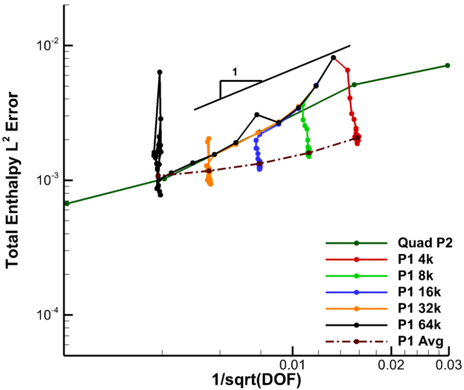
HOW5 CI1 - Adapted Meshes (Enthalpy Error, P=3)



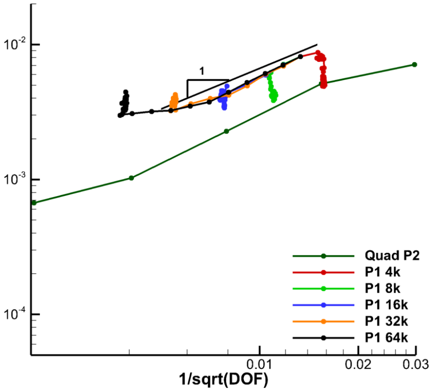
HOW5 CI1 - Adapted Meshes ($\bar{P}_t$, P=1)



Minimize Enthalpy Error



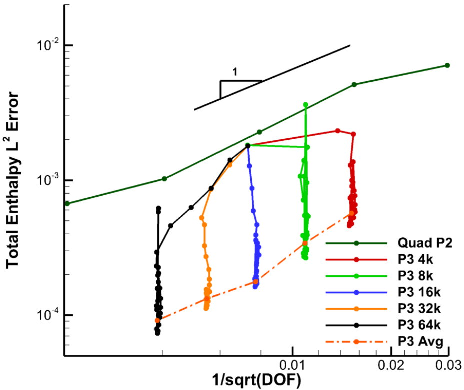
Minimize Total Pressure Error



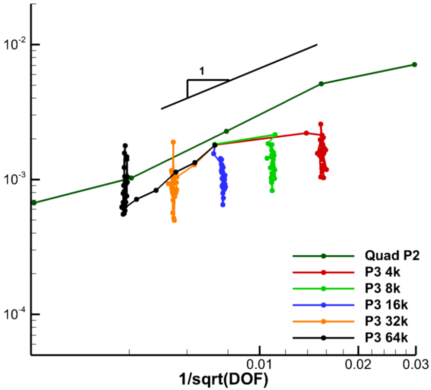
Selective average of 5 converged solutions

HOW5 CI1 - Adaptation History (Enthalpy Error, P=3)

Minimize Enthalpy Error



Minimize Total Pressure Error

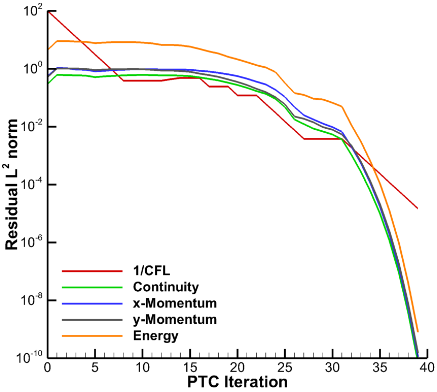


Selective average of 5 converged solutions

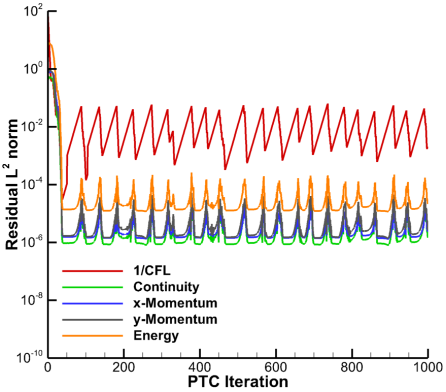
HOW5 CI1 - Pseudo Time Continuation Failure

P3 64k

Grid 6

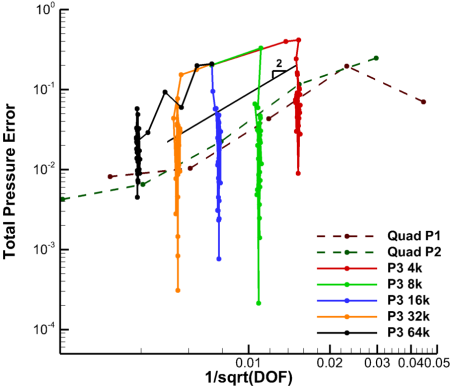


Grid 7

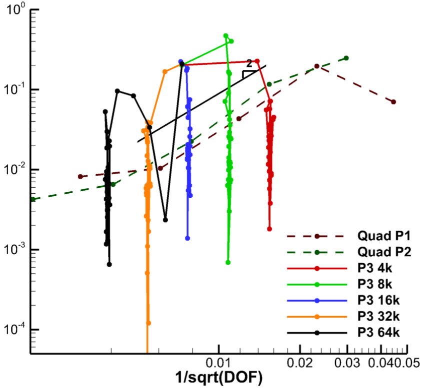


- Non-linear solver?
- Switches in regularization?

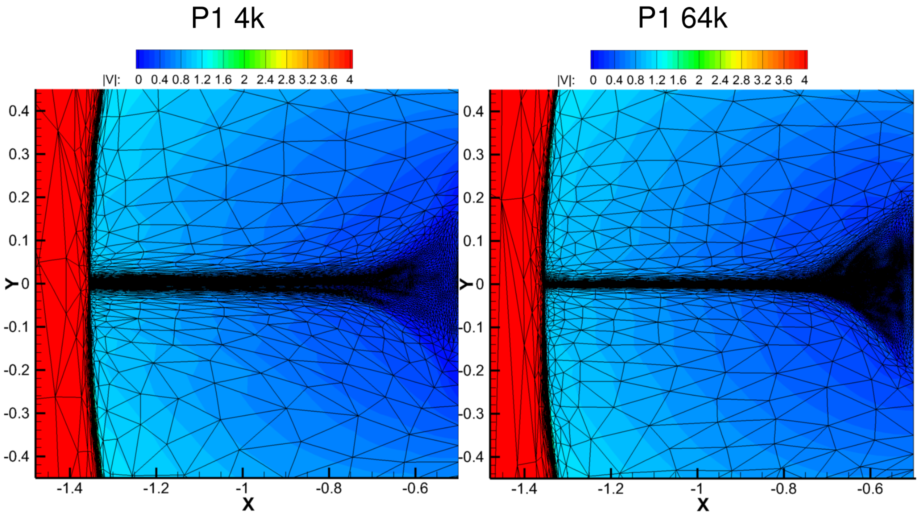
Minimize Enthalpy Error



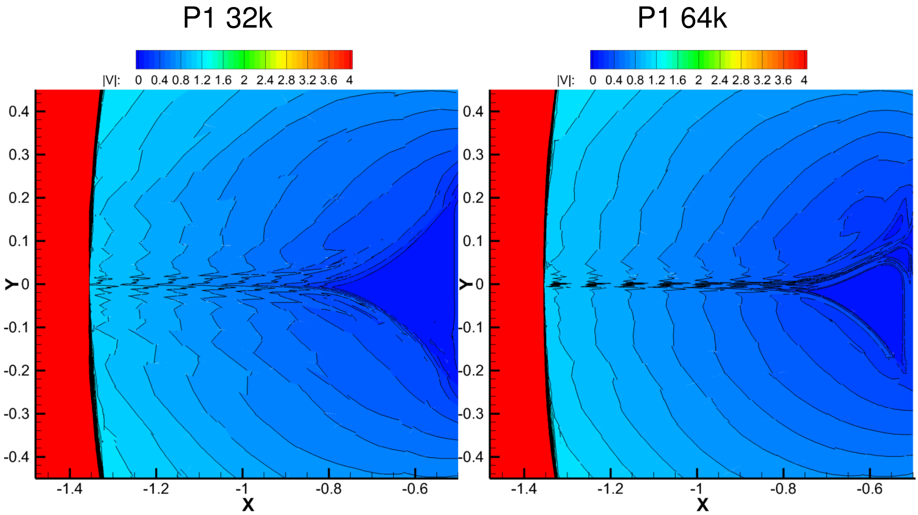
Minimize Total Pressure Error



Minimize Total Pressure Error

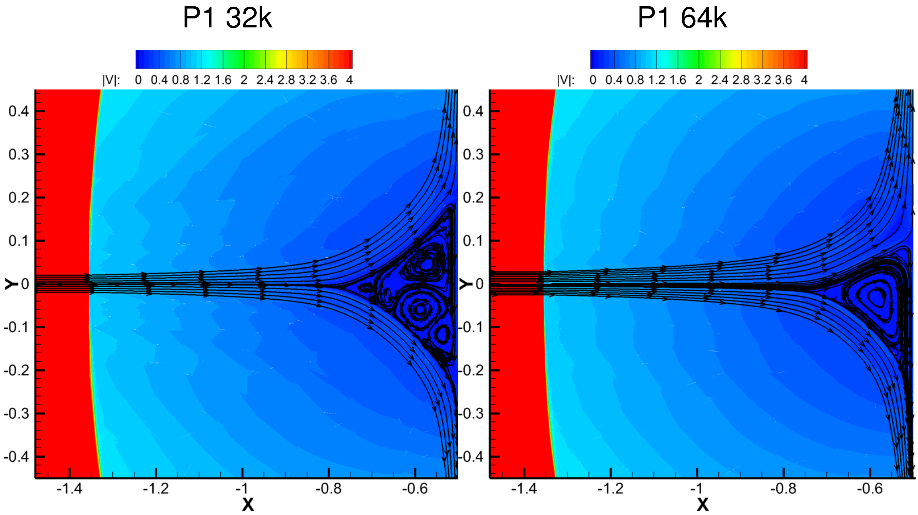


Minimize Total Pressure Error



HOW5 CI1 - Hershey Kiss Stagnation

Minimize Total Pressure Error



Conclusion

- Verification case shows shock capturing only impacts coarse grids
- Total enthalpy error: 1^{st}-order accuracy for all P
- Total pressure error: $\sim 2^{st}$-order accuracy for all P
- Mesh adaptation reduced total enthalpy error (but not rate)
- Total pressure error challenging for asymmetric grids
- Non-linear solver robustness
  - Shock capturing toggling due to switches?
  - Near negative density and temperature during non-linear solve
    - *Surrogate variables*, map from $\mathbb{R}$ to $\mathbb{R}^+$ for density and temperature (c.f log variables)
- Flat region allows stagnation point to float
  - Intentional and good challenge
  - Hershey kiss separation (not present on symmetric grids)

References I

Marshall C. Galbraith, Steven Allmaras, and David L. Darmofal.
A verification driven process for rapid development of cfd software.
AIAA 2015-0818, January 2015.

Garrett E. Barter.
Shock Capturing with PDE-Based Artificial Viscosity for an Adaptive, Higher-Order, Discontinuous Galerkin Finite Element Method.
PhD thesis, Massachusetts Institute of Technology, Department of Aeronautics and Astronautics, June 2008.

Garrett E. Barter and David L. Darmofal.
Shock capturing with PDE-based artificial viscosity for DGFEM: Part I, formulation.
229(5):1810–1827, 2010.

References II

Masayuki Yano.
An Optimization Framework for Adaptive Higher-Order Discretizations of Partial Differential Equations on Anisotropic Simplex Meshes.
PhD thesis, Massachusetts Institute of Technology, Department of Aeronautics and Astronautics, June 2012.

Todd Michal and Joshua Krakos.
Anisotropic mesh adaptation through edge primitive operations.
AIAA 2012-159, 2012.

Backup Slides

Method

- DG-BR2 $\eta = 2 \times$ *Faces* ($\eta = 8$ for Quad grids)
- Quadrature order = $3\,(p+2)-1$
- Q=4 Polynomial Quad Grids
- Pseudo-transient continuation on Newton-Raphson with line search
- Direct linear solve from MKL

We solve,

$$\nabla \cdot \mathbf{F}(\mathbf{u}) = \nabla \cdot \left(\epsilon_s \tilde{\mathcal{A}} \nabla \mathbf{u}\right), \tag{3}$$

where ϵ_s is the artificial viscosity, and $\tilde{\mathcal{A}}$ ensures the viscosity is applied to the total enthalpy, not total energy.
The artificial viscosity is given by,

$$\epsilon_{s,ij} = \frac{H_{ik} H_{kj} \lambda_{max}(\mathbf{u})}{p} \tilde{s}, \tag{4}$$

where $\tilde{s}$ is from a smoothed shock sensor field given by,

$$\nabla \cdot \left(C_2 H^2 \nabla s\right) + \left[\tilde{S}_k(\mathbf{u}) - s\right] = 0 \tag{5}$$

Smoothed sensor $\tilde{s}$ given by,

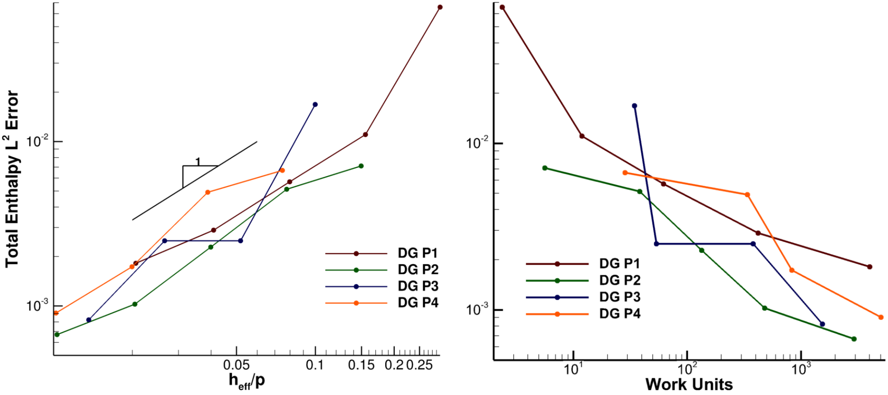
$$\tilde{s} = \begin{cases} 0 & s < \theta_L \\ \frac{1}{2}\theta_H \left(\sin \pi \left(\frac{s-\theta_L}{\theta_H-\theta_L} - \frac{1}{2} \right) + 1 \right) & \theta_L < s < \theta_H \\ \theta_H & s > \theta_H \end{cases} \tag{6}$$

The shock detector is given by,

$$\tilde{S}_k (\mathbf{u}) = f_{switch} \left(\frac{1}{|\partial\kappa|} \int_{\partial\kappa} \left| \frac{[\![g(\mathbf{u})]\!]}{\{ g(\mathbf{u}) \}} \right| dA \right) \tag{7}$$

Here we used the pressure for g.
See Yano PhD dissertation for all details.

L^2 norm of stagnation enthalpy in the domain

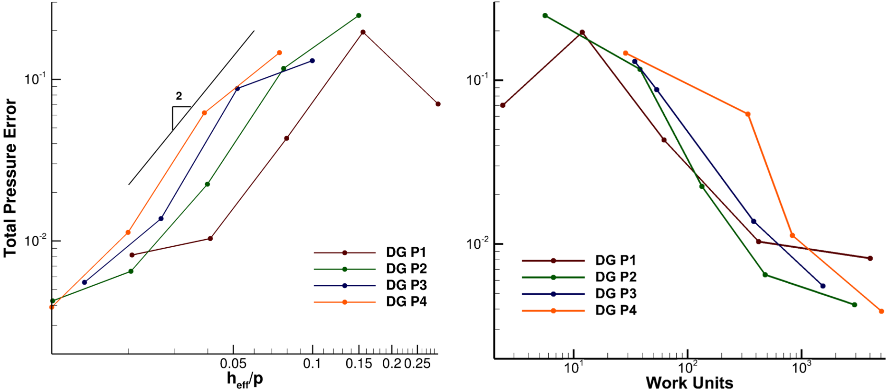


Approx. 1^{st} order convergence.

HOW5 CI1 - Provided Grids

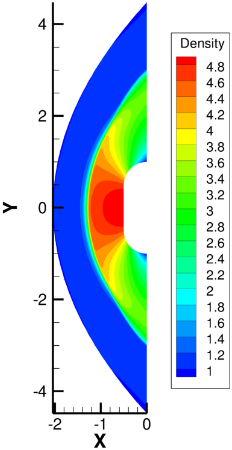
Gaussian weighted ($g(y)$) total pressure error:

$$\bar{P}_t = \int_{\Gamma_w} g(y)\,(P_t(y) - P_{02})\,dy$$

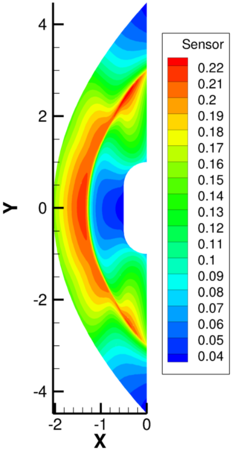


Approx. 2^{nd} order convergence

HOW5 CI1 Case - Grid 0: 2D Euler with artificial viscosity
Mach 4.0



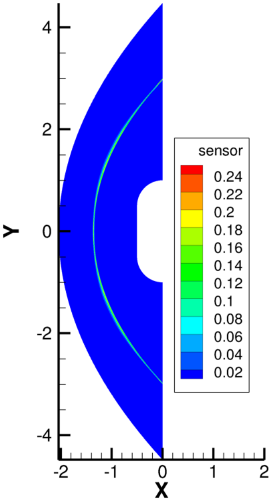
(a) Density - DG(3)

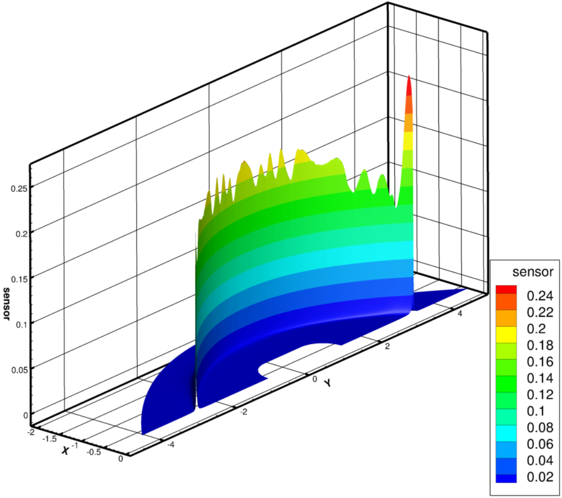


(b) Sensor - DG(3)

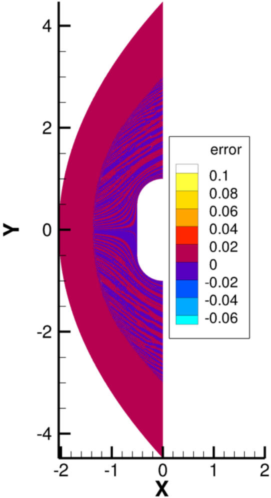
HOW5 CI1 - Provided Grids

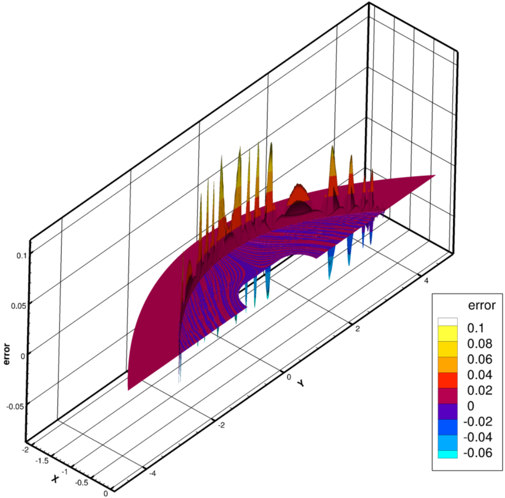
Noisy sensor field around shock



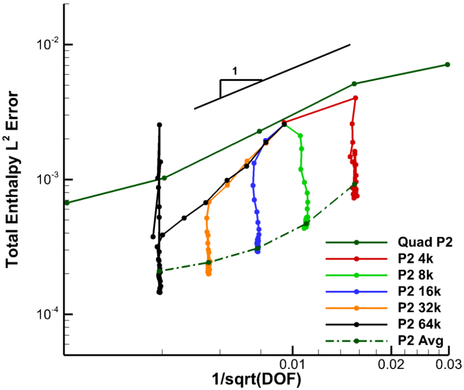


Streaks in stagnation enthalpy error behind the shock

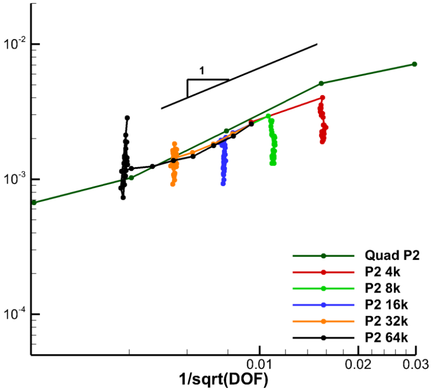




Minimize Enthalpy Error

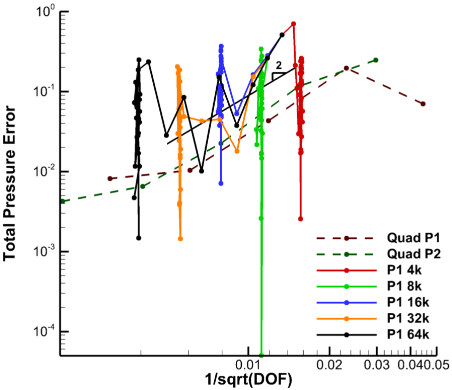


Minimize Total Pressure Error

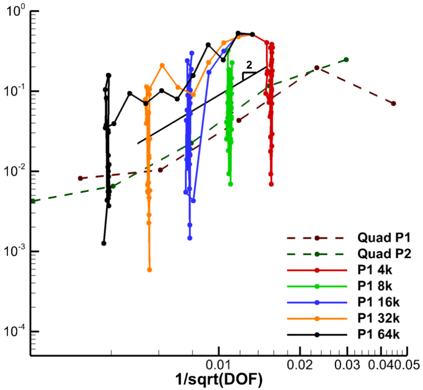


HOW5 CI1 - Adaption History (P_t Error, P=1)

Minimize Enthalpy Error

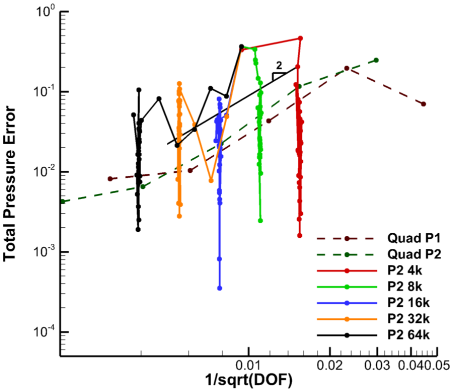


Minimize Total Pressure Error



HOW5 CI1 - Adaption History (P_t Error, P=2)

Minimize Enthalpy Error



Minimize Total Pressure Error