5th High Order CFD Workshop   Case I 1

Inviscid Bow Shock upstream of a blunt body in supersonic flow

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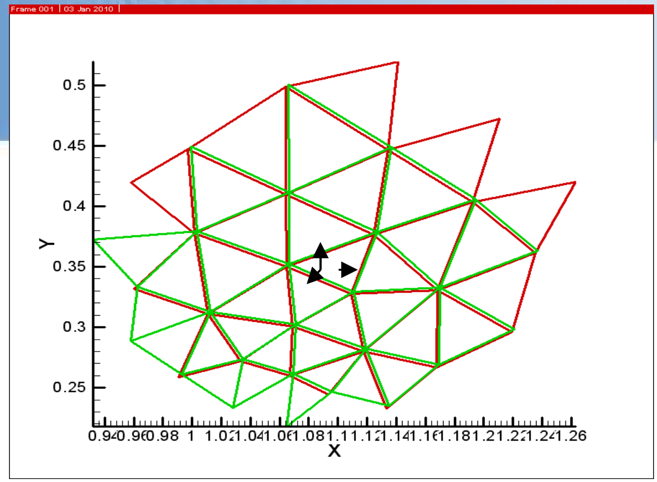
NXO : FV method on unstructured grids Quick reminder



•1 dof per cell and per equation

• Non compact scheme

1/ Polynomial reconstruction of a given field over a (wide) stencil (red, green, union of the 2) by weighted least squares :

$$\phi_a(X,Y) = a_{\{ij\}} X^i Y^j = a_1 + a_2 X + a_3 Y + a_4 X^2 + a_5 XY + a_6 Y^2 + a_7 X^3 + ...$$

Each coefficient a_n is a linear combination of the discrete values of the field (cell means) in the stencil : coefficients computed in the preprocessor

2/ Unlimited projection of the polynomial on the cell interfaces also in the preprocessor : gives the surface mean of the scalar field ϕ , or (new feature) the surface flux integral of a vector field $[F,G]^T$ across a curvilinear interface,

$$\hat{\varphi} = \oint_S (F\upsilon_X + G\upsilon_Y) dS = \sum_{c \in St} \left(\lambda_{X,c} \overline{F}_c + \lambda_{Y,c} \overline{G}_c \right)$$

as a linear combination of the discrete fields

➔ efficient High Order integration after the High Order reconstruction

References : AIAA paper San Diego SciTech 2016 Project “researchgate Le Gouez”

Preliminary validation for regular fields on HO grids and boundaries : inviscid Gaussian Bump, Ringleb flow (3rd HO workshop)

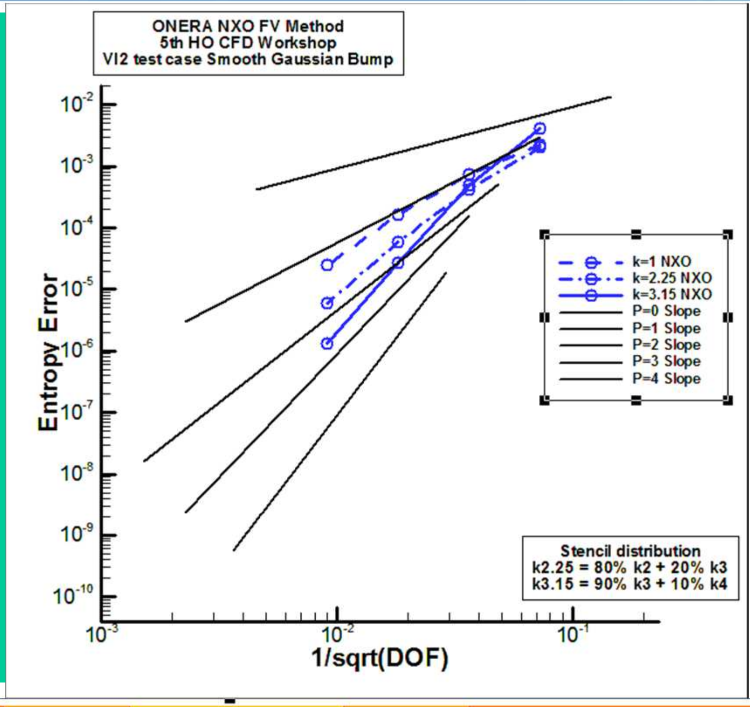
Gaussian bump

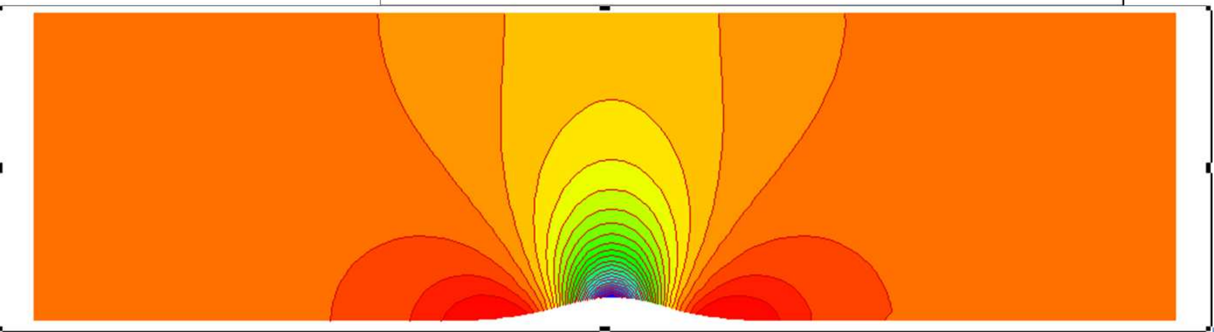
Rms of Entropy error as function of the mesh size

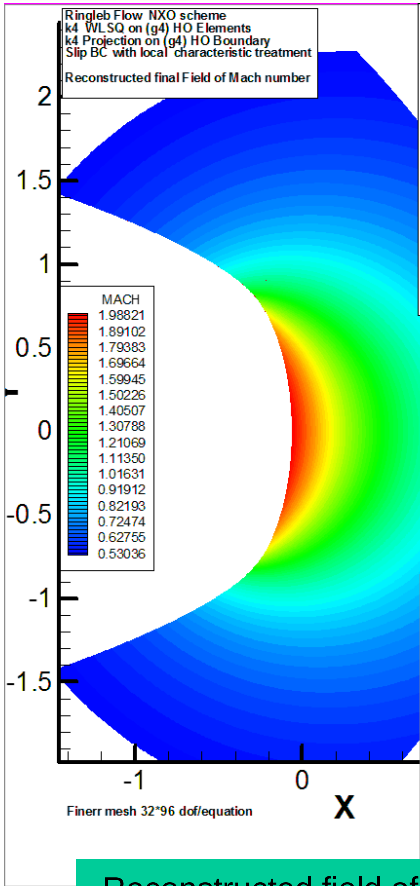
Convergence order **4.1** for the k3+ reconstruction of the flux density tensor components and curvilinear projection

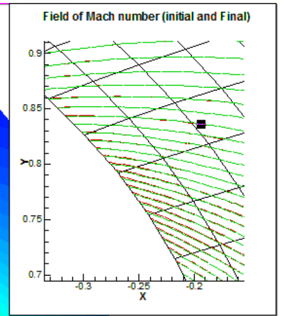
Density field grid 4

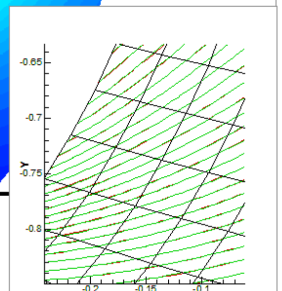
3072 dof/eqn











- Reconstructed field of Mach number
- Exact and converged solutions
- Medium grid 32*96

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NXO Centered scheme with 2nd and 4th order artificial dissipation
HO expression of the sum, first and third differences of the JST scheme

Conservation equations $$\frac{\partial W_i}{\partial t}+\frac{\partial F_i}{\partial x}+\frac{\partial G_i}{\partial y}=0 \qquad W=\left[\rho,\rho u,\rho v,\rho e_t\right]^T$$

$$f_i = F_i \upsilon_x + G_i \upsilon_y$$

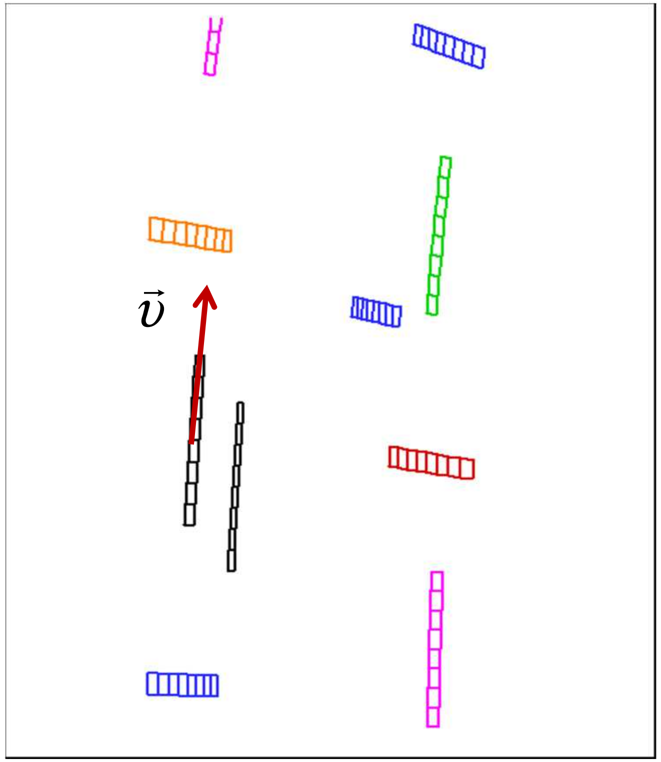
$$\frac{\partial\left(\Omega \overline{W_i}\right)}{\partial t}+\sum_{n=1}^{ni} S_n\left(\hat{f}_{i,nat}-\hat{f}_{i,diss}\right)_n=0$$

$$\hat{f}_{i,nat}=\sum_{s\in St,left\cup St,right}\lambda_s\,\bar{f}_{i,s} \qquad \hat{f}_{i,diss}=\varepsilon_2\,\omega\hat{\delta}W_i^{*}-\varepsilon_4\omega\hat{\delta}\hat{\delta}\hat{\delta}\,W_i^{*}$$

$$W^{*}=\left[\rho,\rho u,\rho v,\rho h_t\right]^T$$

$$\hat{\delta}W_i^{*}=h\int_S\frac{\partial P_i^{*}}{\partial \upsilon}dS=h\sum_{St,left\cup St,right}\mu_{1s}\overline{W}_{i,s}^{*}$$

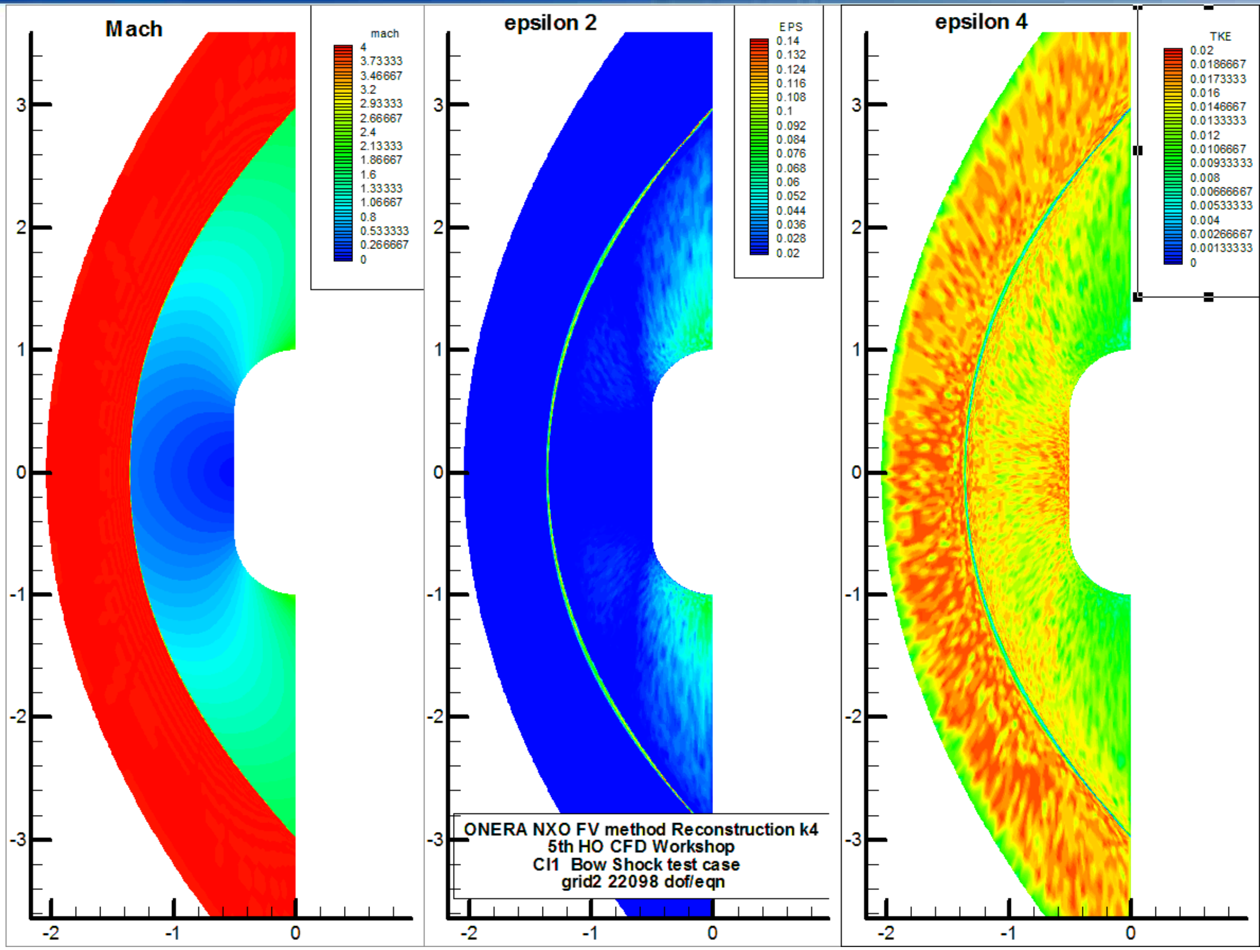
$$\hat{\delta}\hat{\delta}\hat{\delta}W_i^{*}=h^3\int_S\frac{\partial^3 P_i^{*}}{\partial \upsilon^3}dS=h^3\sum_{St,left\cup St,right}\mu_{3s}\overline{W}_{i,s}^{*}$$



1D stencils : 6, 8 and 10-cell stencils have been used : k2, k3, k5

$$\phi_a(\upsilon)=a_1+a_2\upsilon+a_3\upsilon^2+a_4\upsilon^3+a_5\upsilon^4+a_6\upsilon^5+\ldots.$$

All the field values of conservative variables and flux densities in the stencil cells are present in the sum and differences formulas



Iso_Mach and coefficients of artificial dissipation ; discontinuity sensor based on variation of pressure across the stencil

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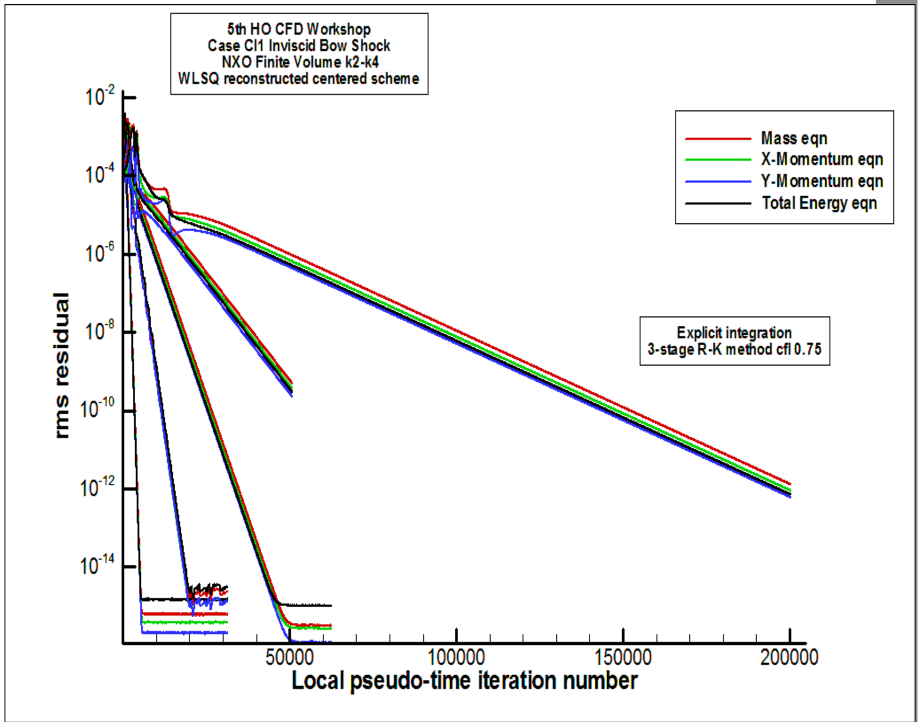
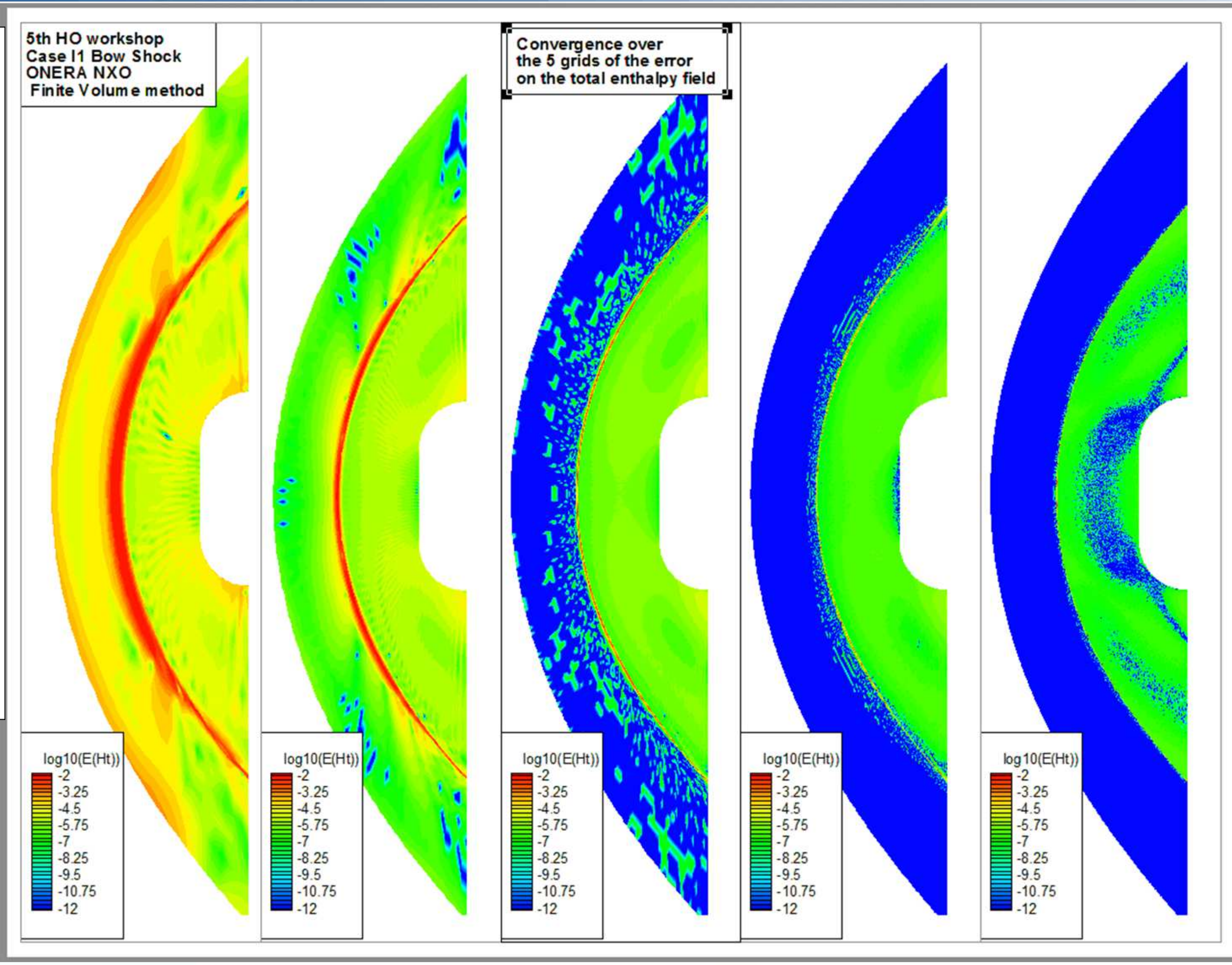
Solution convergence

Total enthalpy error field





Explicit integration by RK3 (Shu-Osher)

Convergence of the solution

Fields of local error in total enthalpy (log10)

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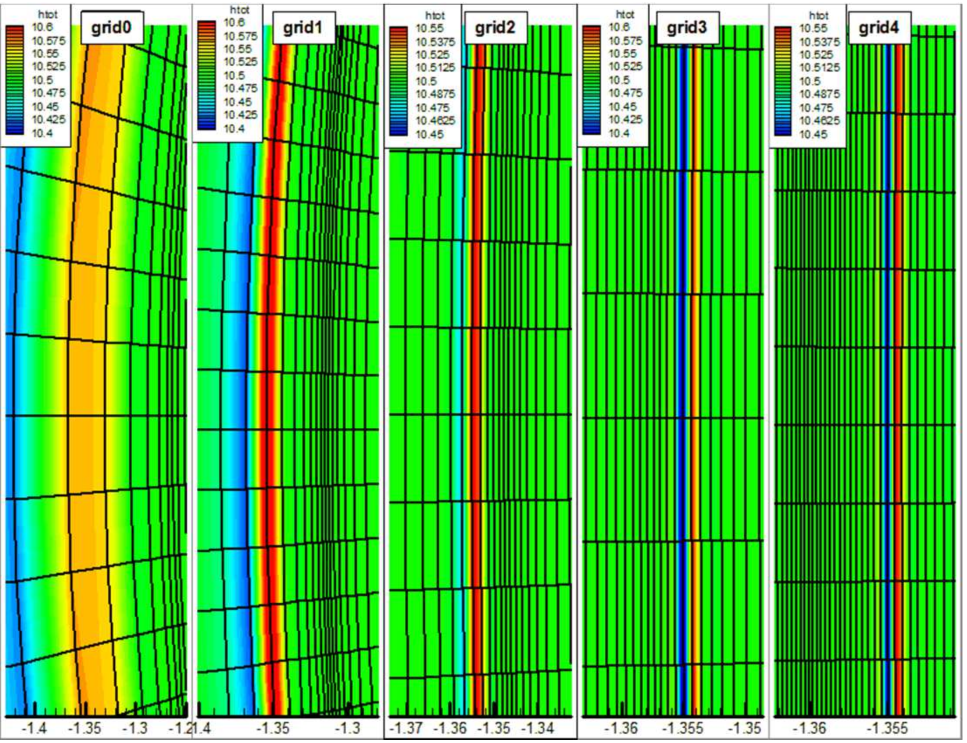
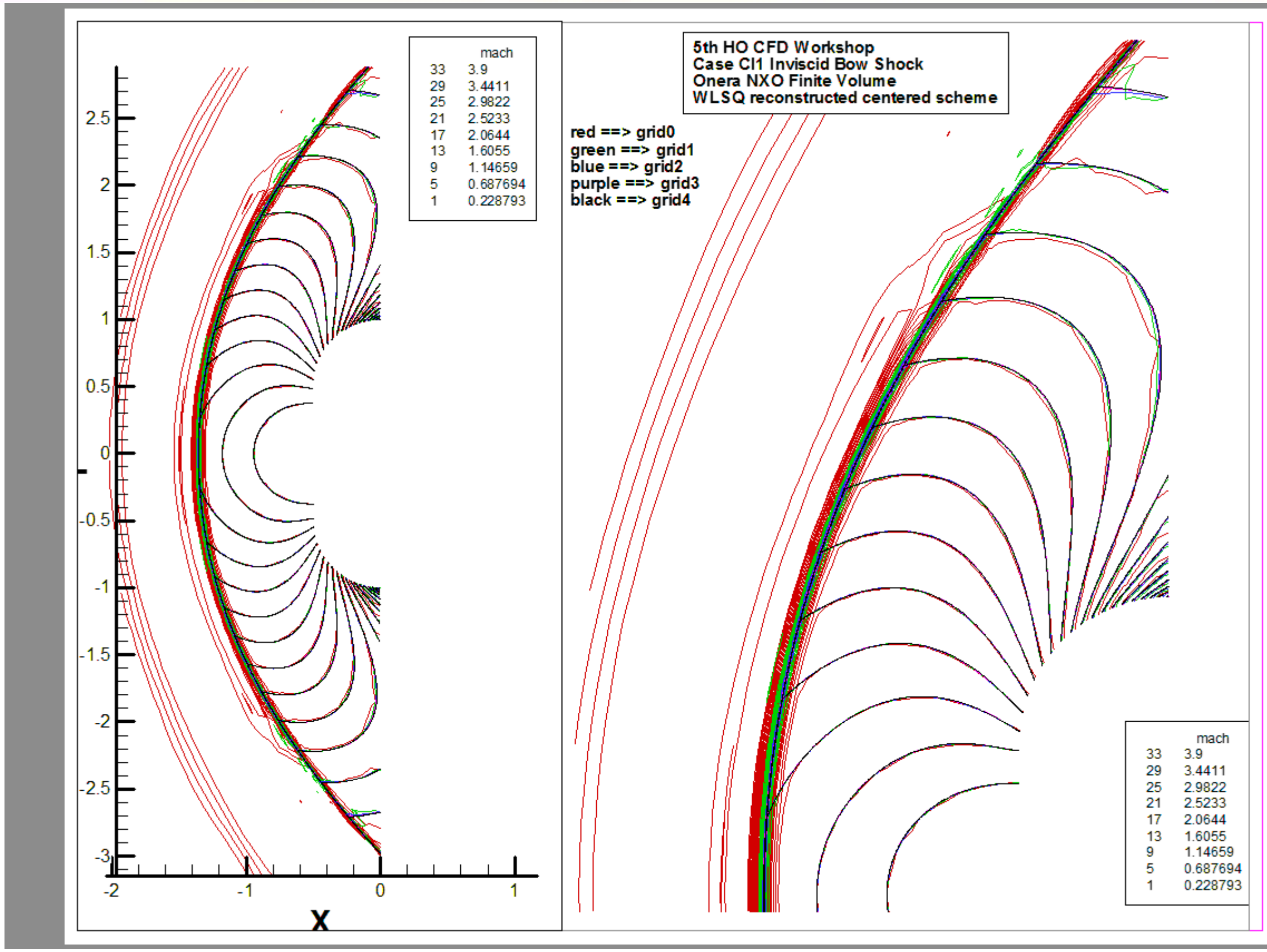


Shock location with respect to the grid refinement
Field of total enthalpy



Iso-Mach lines in the 5 solutions

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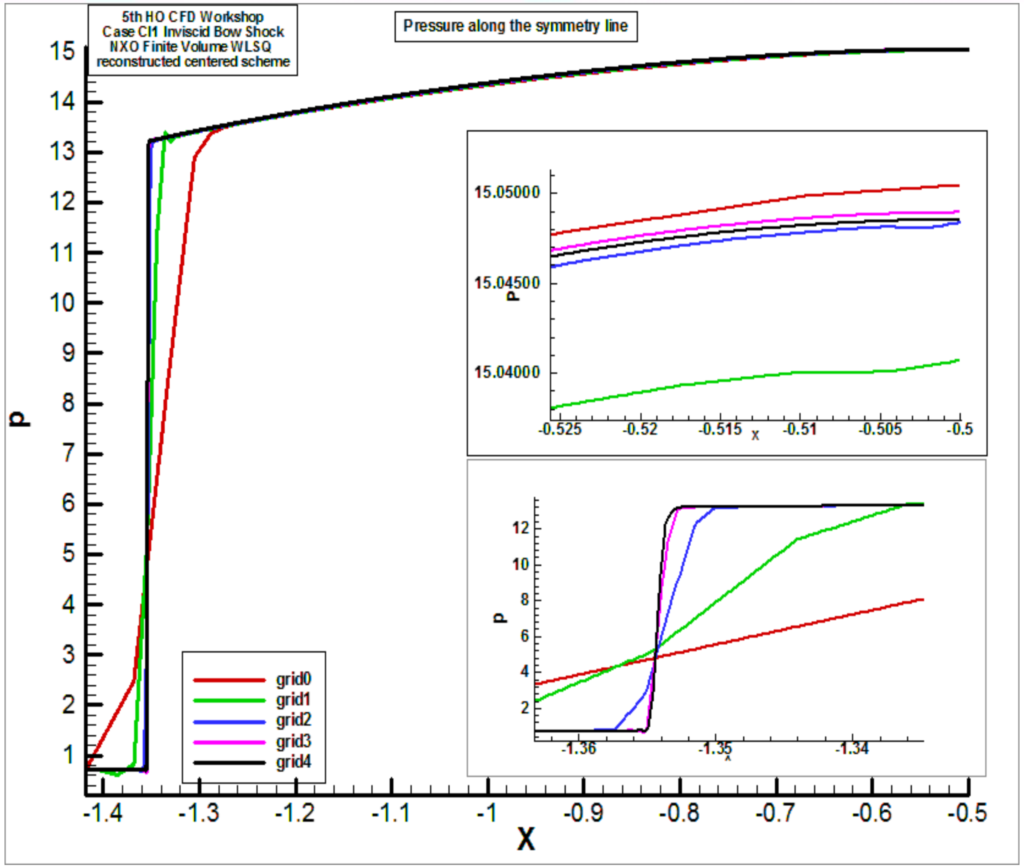

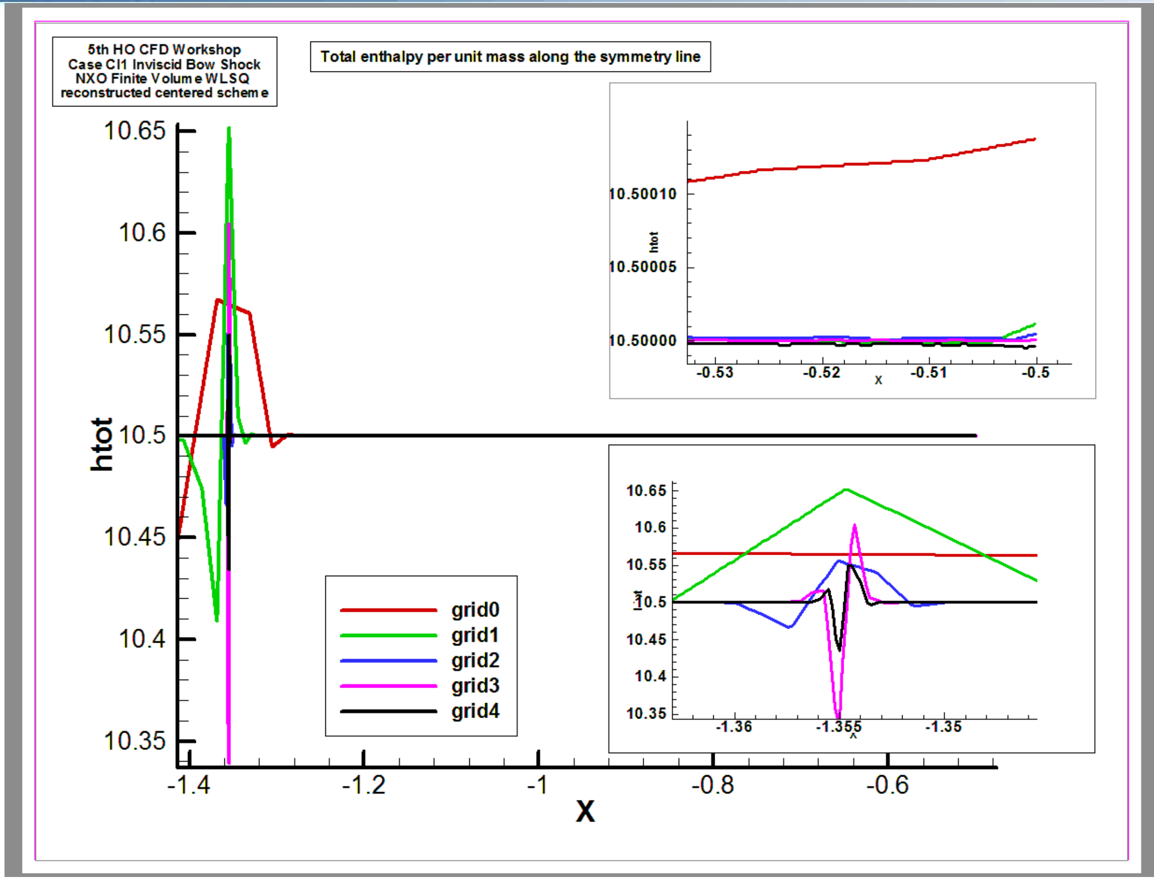

Pressure and total enthalpy along the symmetry line
oscillation of total enthalpy of the order of 1 to 2 %

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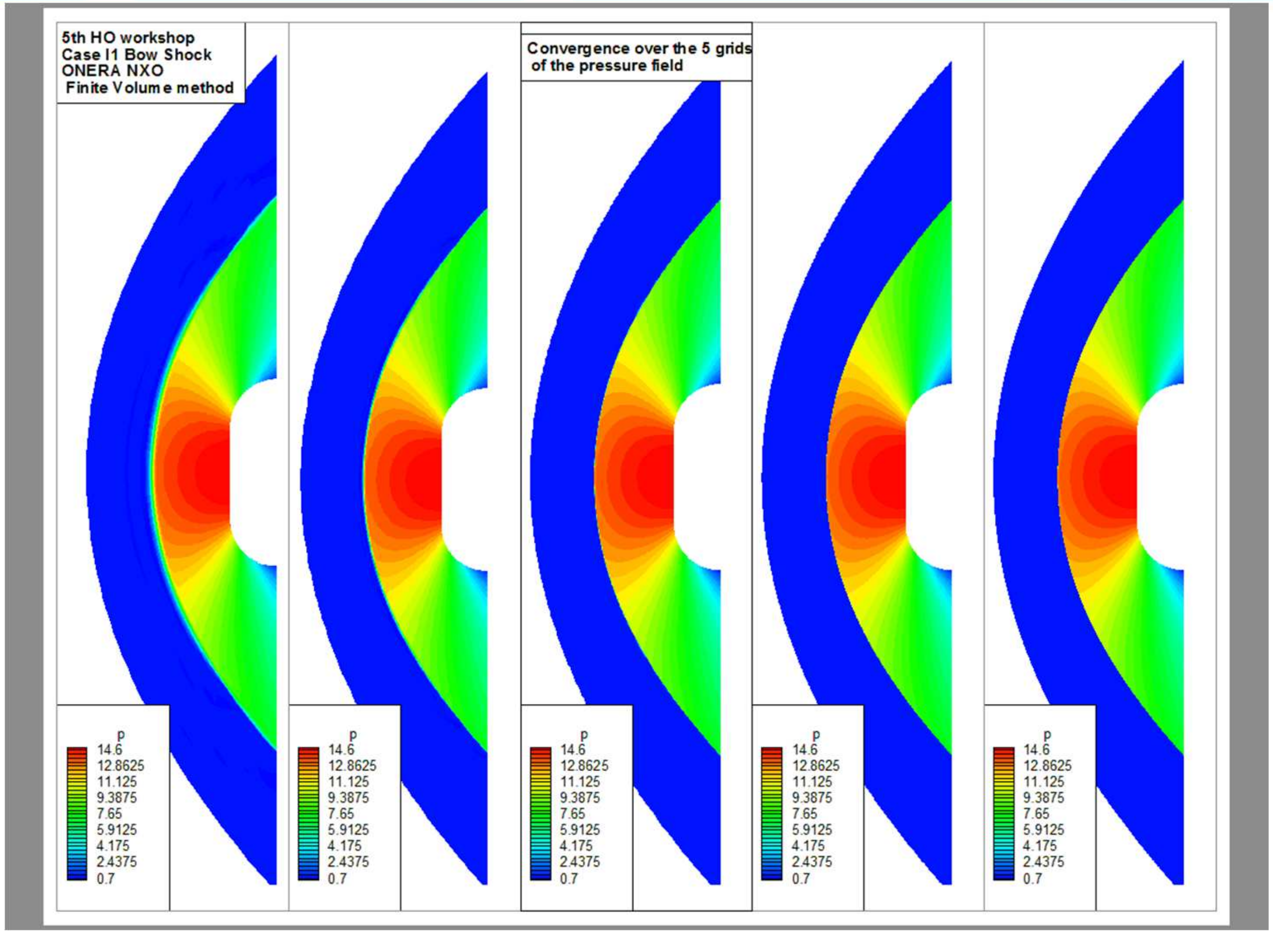
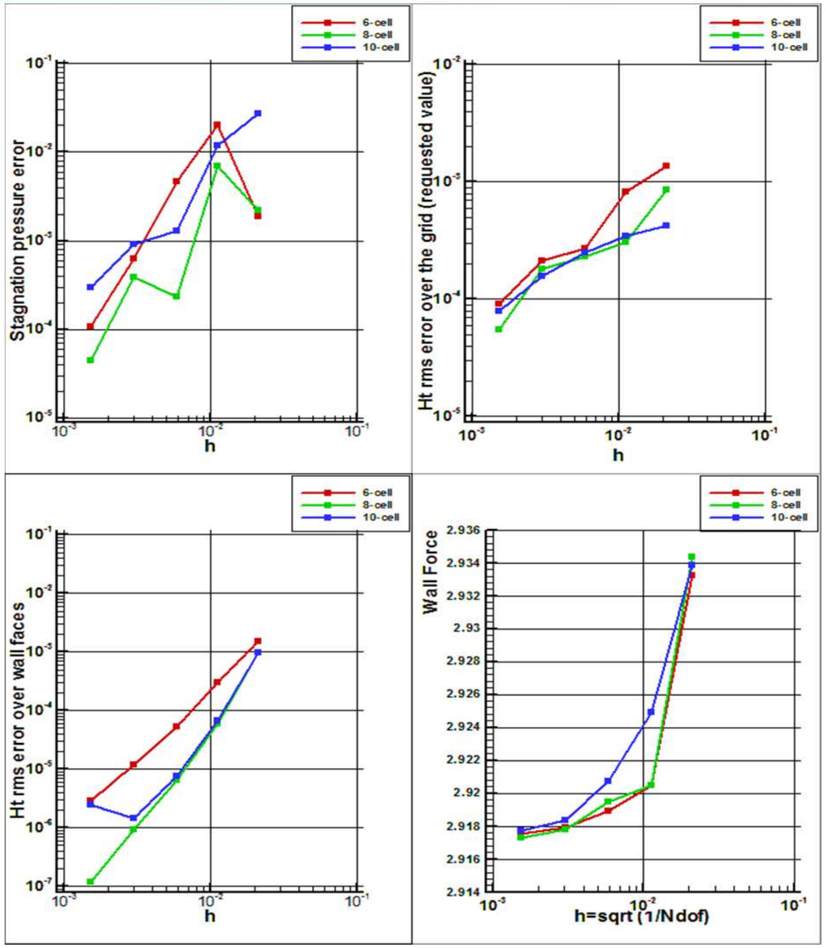
Pressure field



Space convergence orders



Convergence of the error indicators and force on wall

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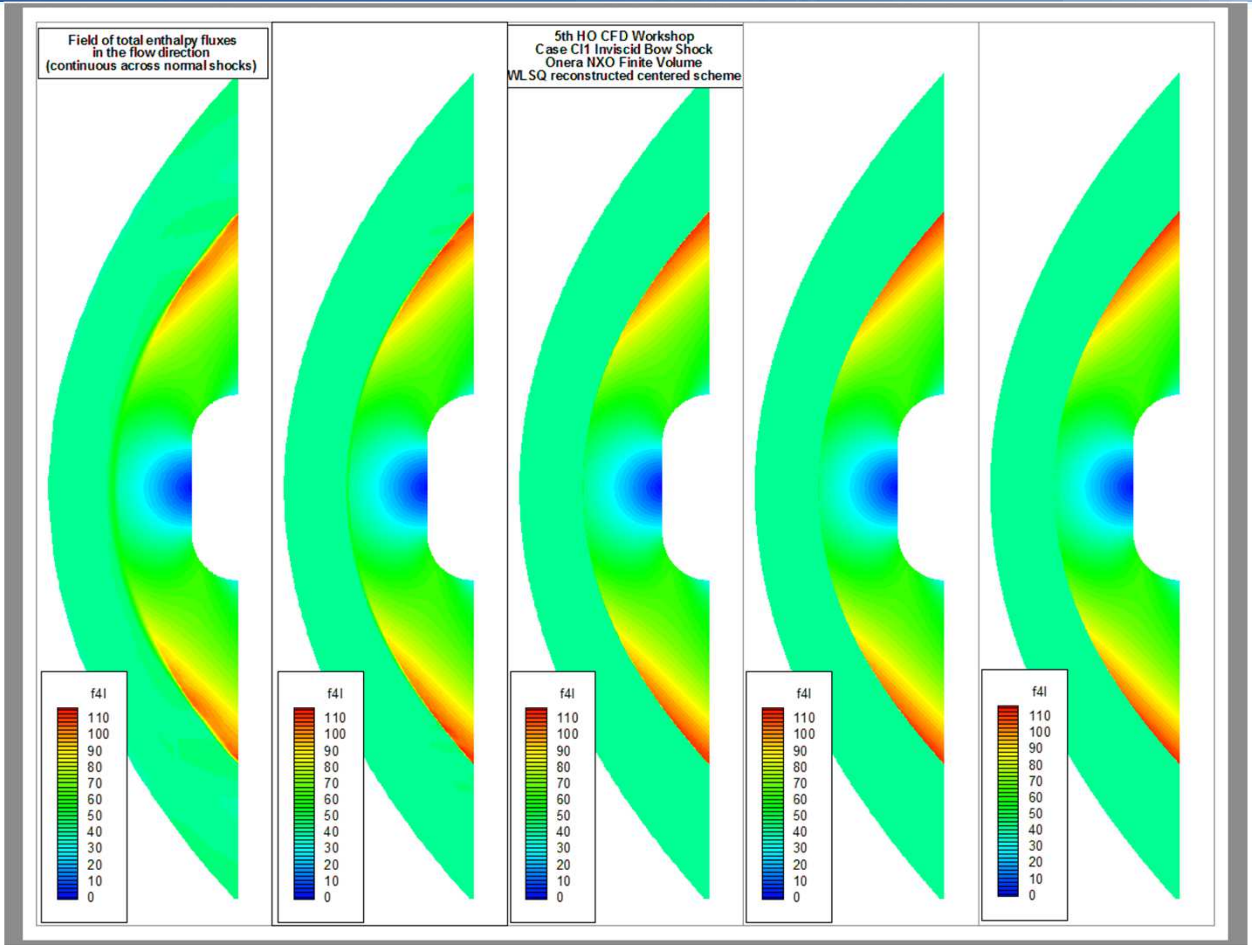


Field of fluxes in the flow direction for the energy equation (quantity interpolated at HO in the stencils across the normal shock

Conclusion and perspectives

New scheme variant is accurate and stable across shocks
The 2D version should be used near the walls to improve the wall BC in the tangential direction and its curvature

An extension of this test case could be :

- Computation on a series of triangular grids refined at the wall and at the shock location or not to measure the loss of accuracy for grids not adapted in size and orientation with respect to these.
- Monitor the space convergence order of the x-Force on the flat and circular part of the wall
- Imagine an unsteady version with the wall oscillating in rotation by a few degrees

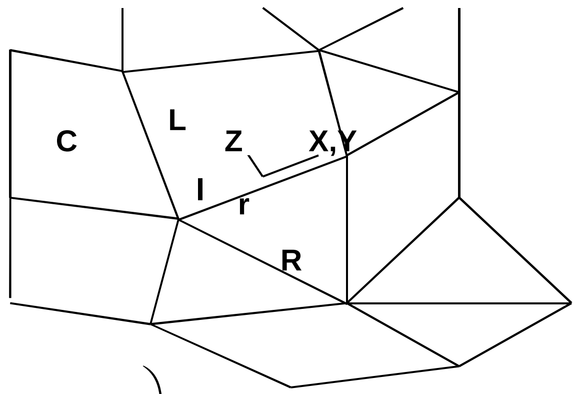
THANK YOU for your attention

FV NXO method : Reconstruction and projection

$$\phi_a(X,Y,Z) = a_{\{ijk\}} X^i Y^j Z^k$$

$$\psi = \sum_{s=1}^{ns} \varpi_s \left(\Omega_{c(s)} \overline{\phi}_{c(s)} - \int_{\Omega_{c(s)}} \phi_a(X,Y,Z)\,dV \right)^2$$

Reconstruction error functional   ns = Stencil size : nb of monomials + 50%



$$\hat{\phi}_l = \sum_{c=1}^{ns} \lambda_c \overline{\phi}_c$$

$$\hat{\vec{\nabla}} \phi_l = \sum_{c=1}^{ns} \vec{\mu}_c \overline{\phi}_c$$

1/ Reconstruction in Left stencil centred on L

$$\frac{\partial \psi}{\partial a_{\{ijk\}}} = \sum_{s=1}^{ns} \varpi_s \left(-2\Omega_{c(s)} \Re^{ijk}_{c(s)} \overline{\phi}_{c(s)} + 2 {\Re^{ijk}_{c(s)}}^2 a_{\{ijk\}} + 2\Re^{ijk}_{c(s)} \sum_{\{ijk\} \neq \{i'j'k'\}} \Re^{i'j'k'}_{c(s)} a_{\{i'j'k'\}} \right) = 0$$

$$\Re^{ijk}_c = \int_{\Omega_c} X^i Y^j Z^k \, dV$$

$$\mathrm{M}_{\{ijk\},c(s)} \overline{\phi}_{c(s)} + \mathrm{A}_{\{ijk\},\{i'j'k'\}} a_{\{i'j'k'\}} = 0 \quad ==> \quad a_{\{ijk\}} = \kappa_{\{ijk\},c(s)} \overline{\phi}_{c(s)}$$

Volume moment of order ijk

2/ Projection on the interface

$$\hat{\phi}_L = \frac{1}{S_{LR}} \int_{\partial\Omega_{LR}} \phi_a(X,Y,Z)\,dS = \frac{\int_{\partial\Omega_{LR}} X^i Y^j Z^k dS}{S_{LR}} a_{\{ijk\}} = \nu_{\{ijk\}} a_{\{ijk\}}$$

$$\hat{\vec{\nabla}} \phi_L = \frac{1}{S_{LR}} \int_{\partial\Omega_{LR}} \vec{\nabla} \phi_a(X,Y,Z)\,dS = \left(i\,\nu_{\{i-1\,jk\}} \vec{e}_X + j\,\nu_{\{ij-1k\}} \vec{e}_Y + k\,\nu_{\{ijk-1\}} \vec{e}_Z \right) a_{\{ijk\}} = \vec{\eta}_{\{ijk\}} a_{\{ijk\}}$$

$$\hat{\phi}_L = \nu_{\{ijk\}} a_{\{ijk\}} = \nu_{\{ijk\}} \kappa g_{\{ijk\},c} \overline{\phi}_c = \lambda \overline{\phi}_c$$

$$\hat{\vec{\nabla}} \overline{\phi}_L = \vec{\eta}_{\{ijk\}} a_{\{ijk\}} = \vec{\eta}_{\{ijk\}} \kappa g_{\{ijk\},c} \overline{\phi}_{c(s)} = \vec{\mu} \overline{\phi}_c$$