U.S. NAVAL RESEARCH LABORATORY

5th International Workshop on High-Order CFD Methods: CI1 - Inviscid bow shock

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DG-FEM Code Overview

- Dimensions:
  - 1-4
- Cell shapes:
  - Simplex, Cuboid, Prism
- Arbitrary polynomial degree:
  - 0, 1, 2, 3, 4, …
  - Isoparametric, subparametric, superparametric,
- Numerical Integration:
  - Project fluxes to a polynomial space
  - Integrate variational form exactly
- Nodes:
  - Equispaced, Gauss-Lobatto

- Standard DG Formulation
  - Convective Flux:
    - Roe, Rusanov, HLLC, Exact
  - Diffusive Flux:
    - SIPG, BR2

- Time Integration:
  - Explicit RK
  - Steady-Newton (with Backward-Euler)
  - Spacetime
- Linear Solvers:
  - GMRES, CG, MINRES, LSQR, …
  - Multigrid in progress
- Nonlinear Solvers:
  - Newton
  - Gauss-Newton
  - Levenberg-Marquardt
  - KKT/Lagrange-Newton
- Preconditioners:
  - Diagonal, Mass, Polynomial
- Parallelism
  - MPI, OpenMP, TBB, CUDA
- Physics
  - Linear Advection, Burgers, Compressible, Incompressible, Reacting Navier-Stokes

MDG-ICE Discretization Scheme

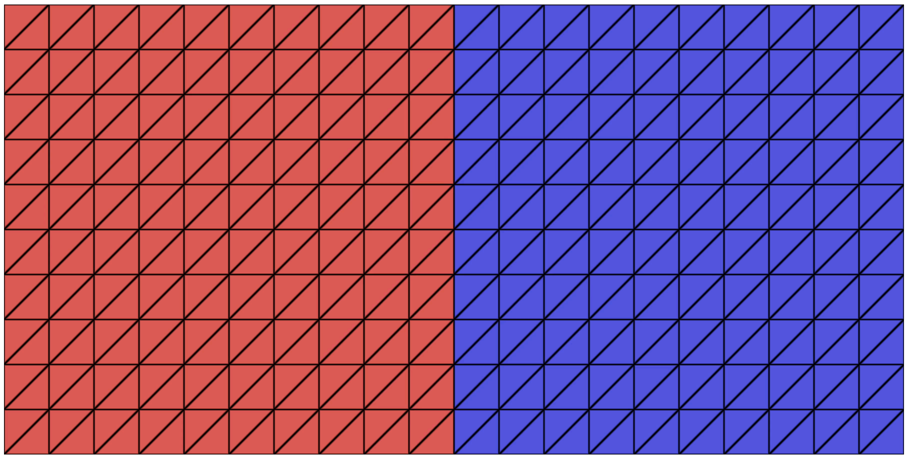
Moving Discontinuous Galerkin with Interface Condition Enforcement

- Detect and solve for interfaces
  - Enforce conservation law and interface condition separately, within a unified finite element weak formulation
  - Solve for flow and grid simultaneously
- Avoids shock capturing
  - Preserves a high-order accurate representation
- General:
  - Dimensionality
  - Steady, unsteady flow
  - Physics
  - Curved interface geometry
  - Non-trivial interface topology
- Not just for shocks/interfaces:
  - Move the grid to resolve the flow, e.g., boundary layers

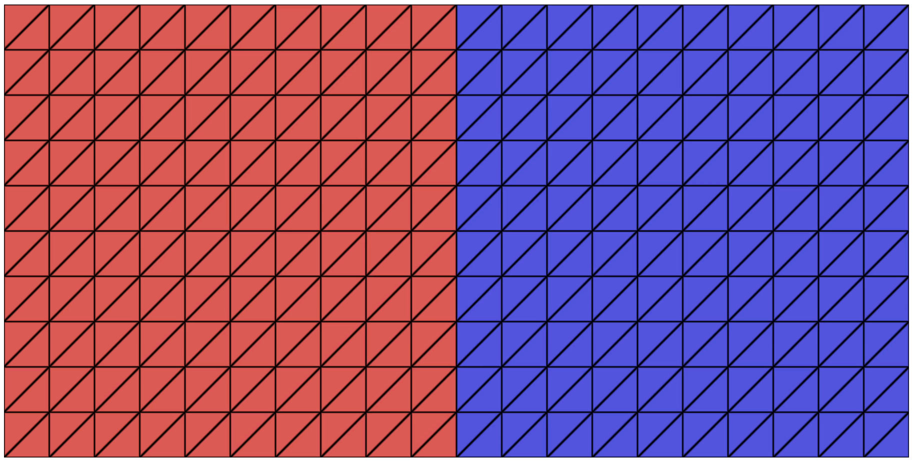
MDG-ICE Discretization Scheme

Moving Discontinuous Galerkin with Interface Condition Enforcement

2D Steady Linear Advection



Velocity = (0.1, 1)



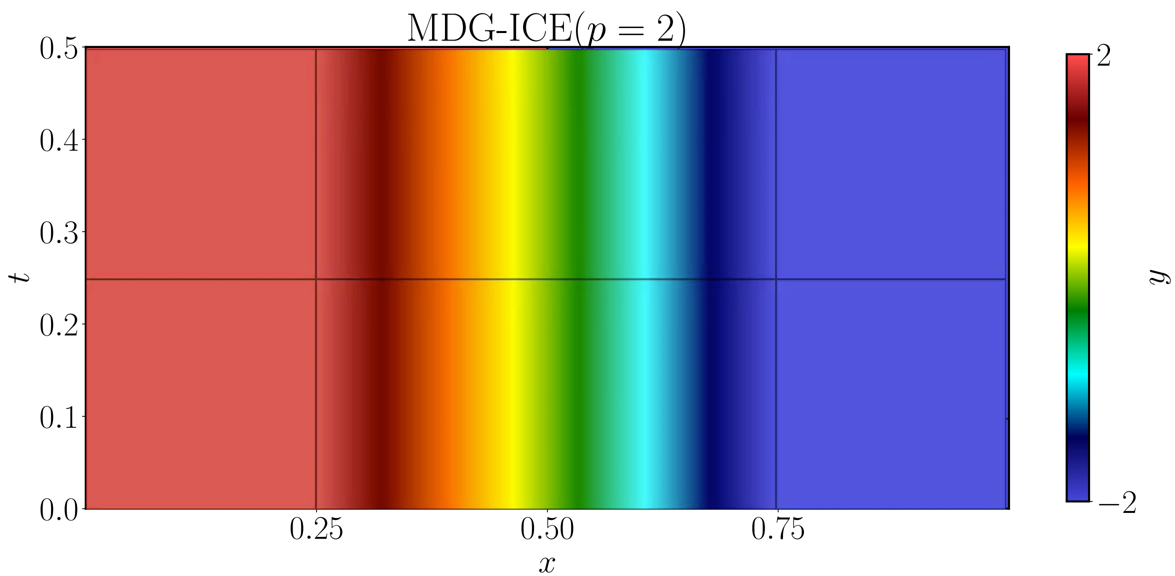
Velocity = (1.5, 1)

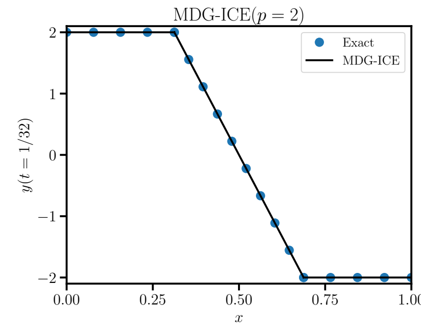
- **MDG-ICE solves for the flow field and grid simultaneously**

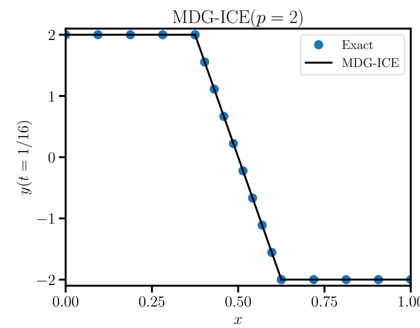
MDG-ICE Discretization Scheme

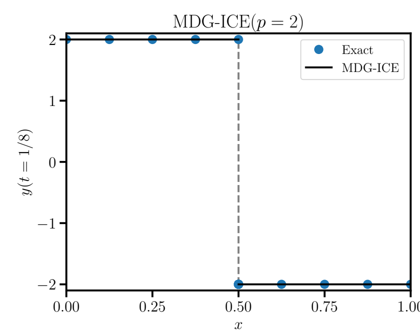
Moving Discontinuous Galerkin with Interface Condition Enforcement

Spacetime Burgers Shock Formation







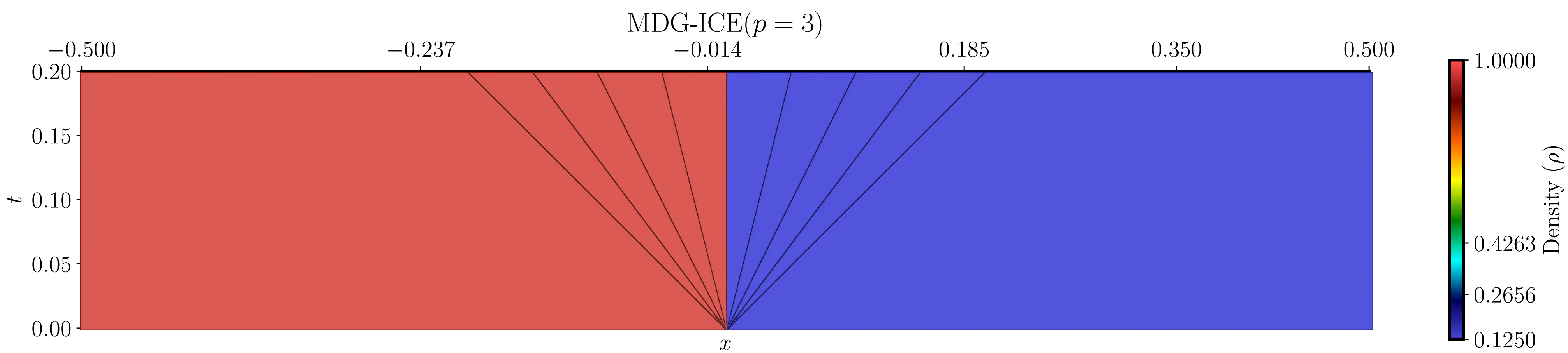


- **MDG-ICE handles:**
  - Unsteady flow via a spacetime formulation
  - Dynamic interface topology
  - Singularities, in conjunction with cuboid degeneration

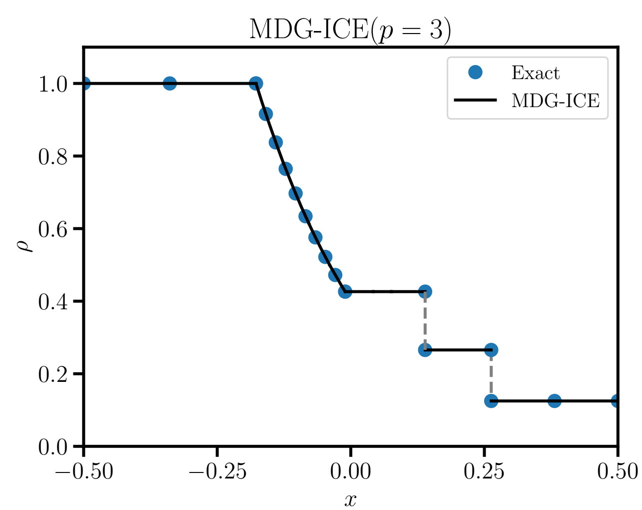
MDG-ICE Discretization Scheme

Moving Discontinuous Galerkin with Interface Condition Enforcement

Sod Shock Tube



- **MDG-ICE handles different types of discontinuities:**
  - Shock, contact, derivative discontinuities

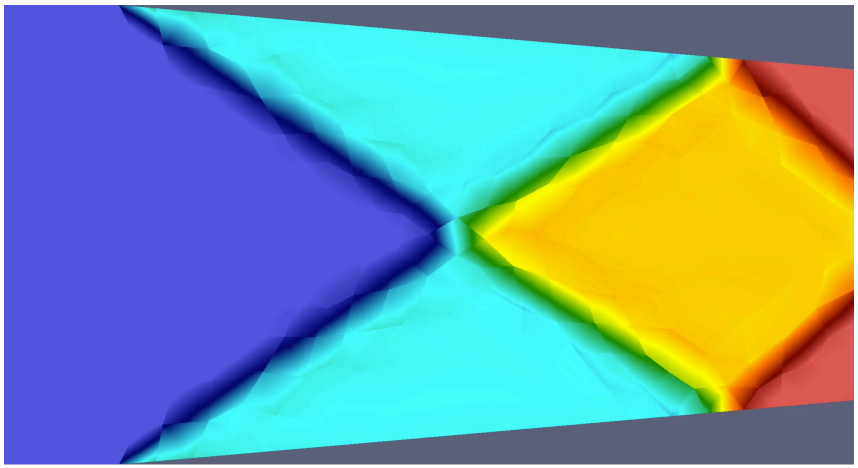


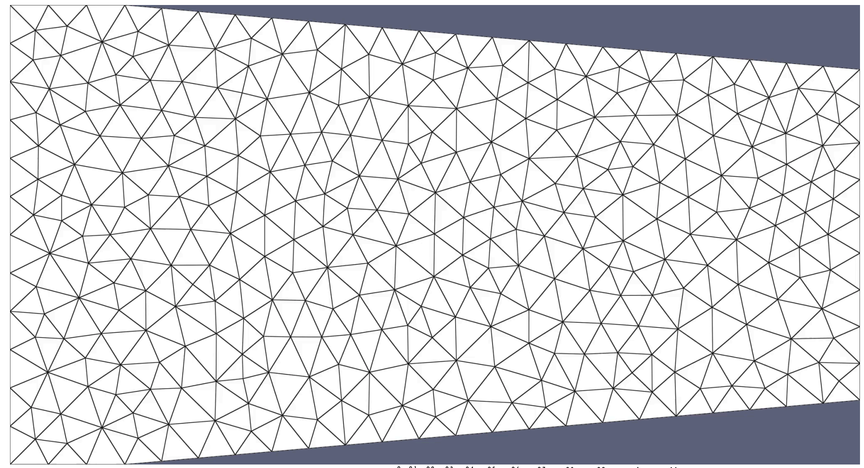
Density at $t = 0.15$

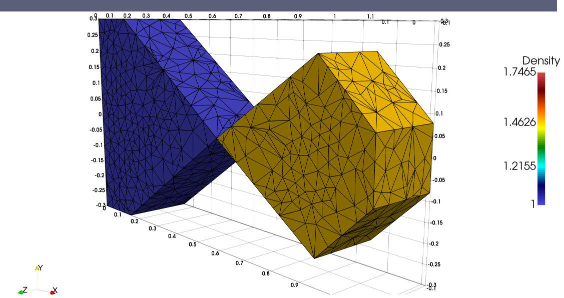
MDG-ICE Discretization Scheme

Moving Discontinuous Galerkin with Interface Condition Enforcement

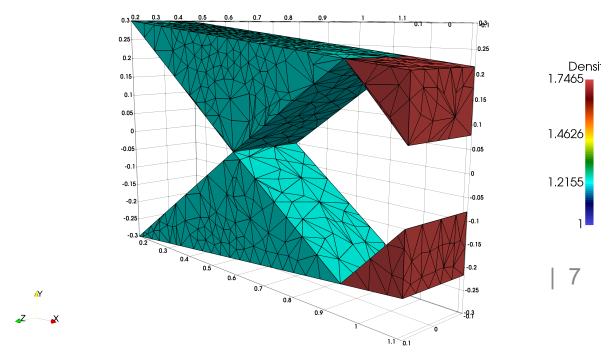
Intersecting Oblique Shocks







- **MDG-ICE handles non-trivial interface topology:**
  - Using standard edge collapse / refine grid operations
    - Also works for higher-dimensional simplexes



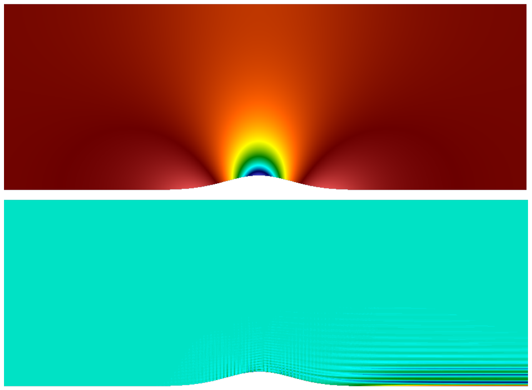
Smooth Bump Verification Case

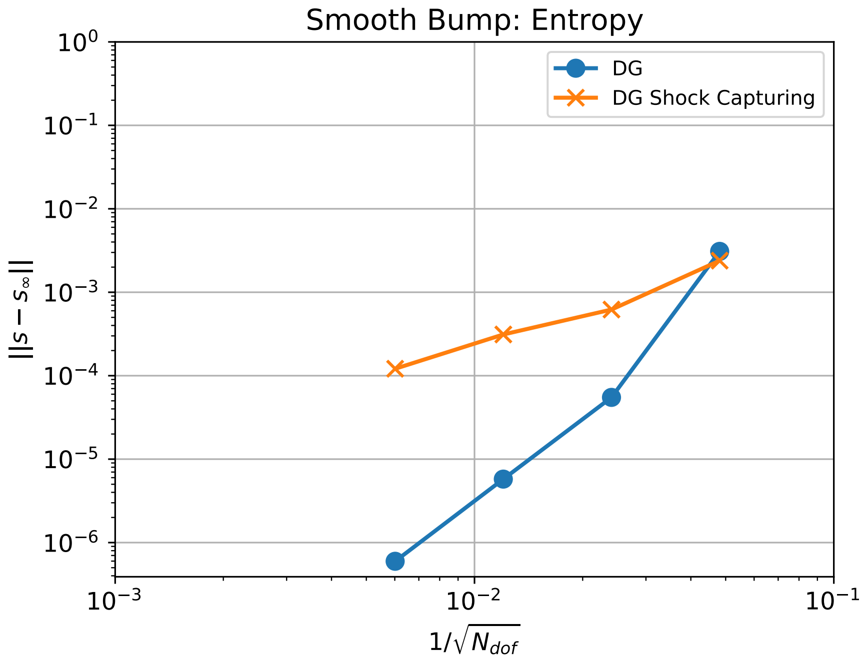
Smooth Bump

- Isoparametric q2 quadrilateral elements
  - Flux projected to q4
- DG
  - Implicit solver: Newton/Backward-Euler
  - Expected order of accuracy observed (slope = 3)
  - Slopes:
    - 5.81, 3.25, 3.27
- DG with Shock Capturing
  - Explicit solver: RK2
  - Slopes:
    - 1.95, 0.99, 1.36

Pressure

Entropy

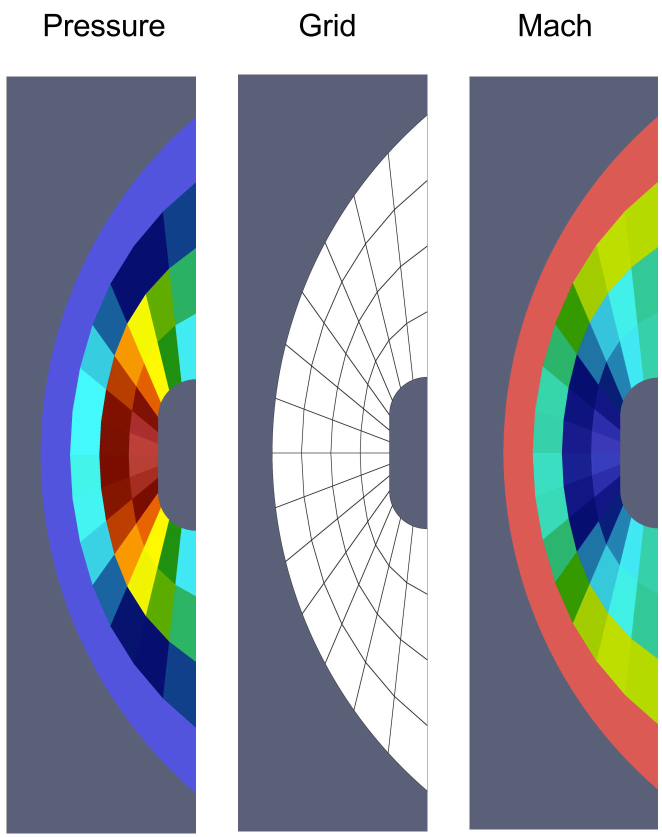




Bow Shock Test Case

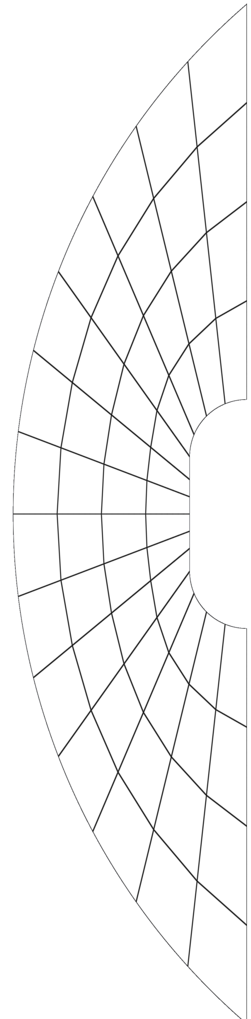
Mach 4 Bow Shock

- Isoparametric q2 quadrilateral elements
  - Flux projected to q4
- DG with Shock Capturing
  - Explicit solver: RK2
  - Residual based shock capturing
- MDG-ICE
  - No shock capturing

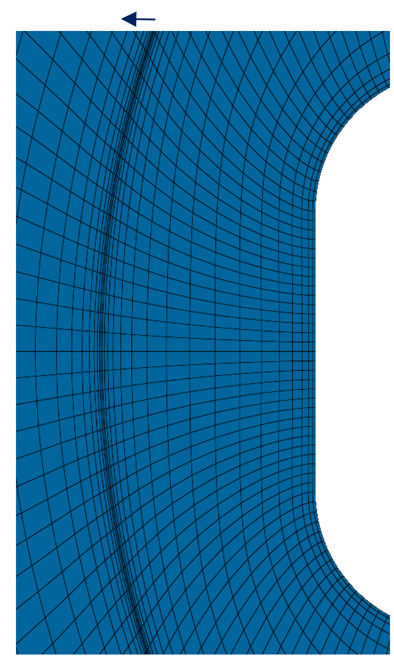


Bow Shock: Geometry

- Ran on uniform structured grids
  - Shock location appeared to be moving across different levels of the provided grids
- Blunt body geometry is exactly as specified
- Inflow surface is modified slightly:
  - Circular arc:
    - Origin 3.85
    - Radius 5.9
    - Top/bottom at:
    - (x,y)=(0, ±4.4704)
- MDG-ICE requires geometric boundary conditions:
  - So that the geometry can move along the boundary



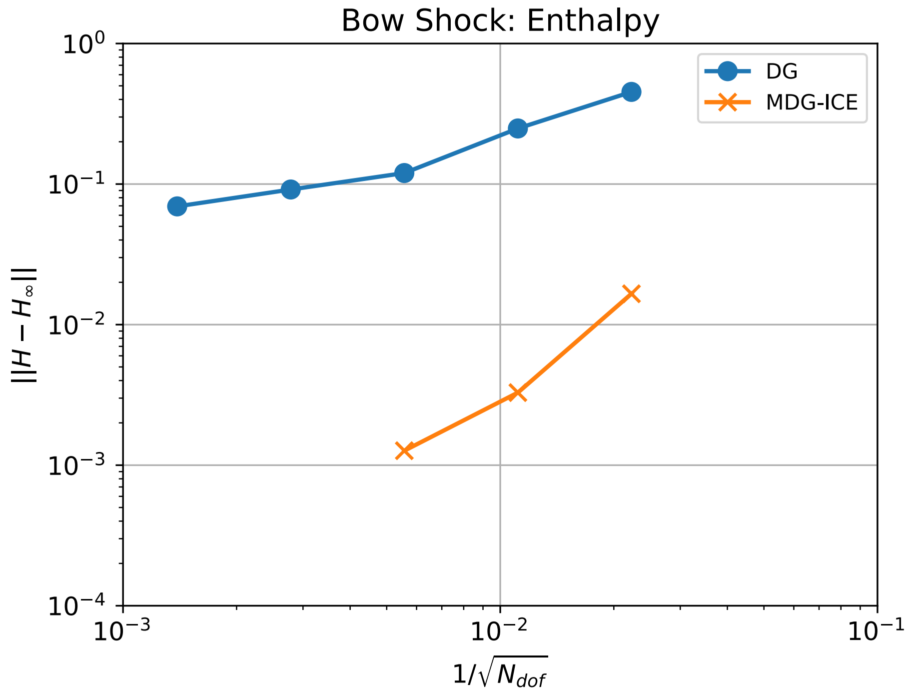
The shock refinement location appears to move on the provided grids:



Bow Shock: Stagnation Enthalpy

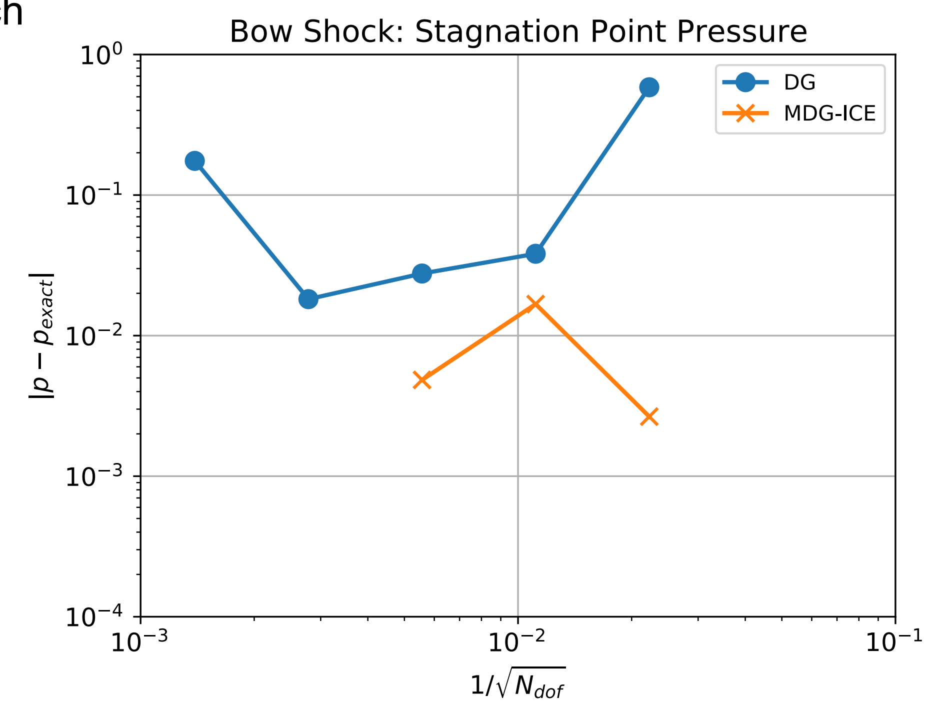
- DG(q=2) isoparametric quadrilateral elements with shock capturing
  - Slopes: 0.86, 1.05, 0.39, 0.40
- MDG-ICE(q=2) isoparametric quadrilateral elements
  - Slopes: 2.34 1.37

Expected slope = 3



Bow Shock: Pressure at the Stagnation Point

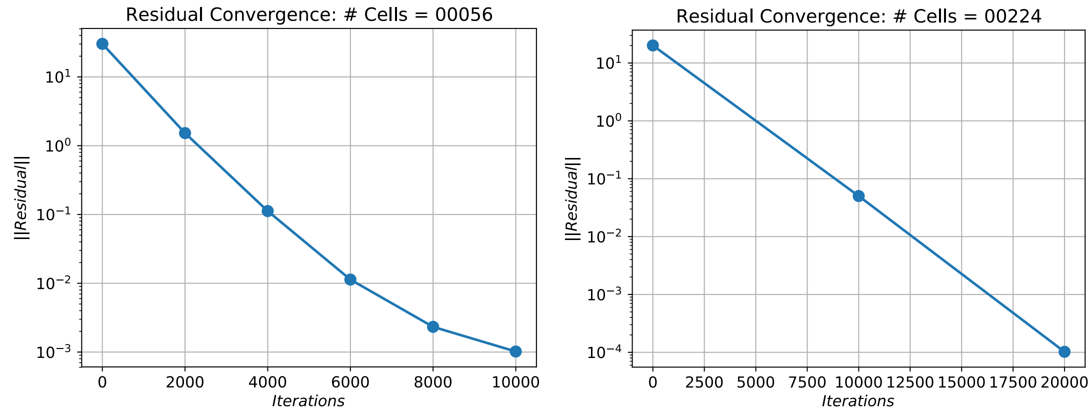
- DG(q=2) isoparametric quadrilateral elements with shock capturing
  - Instabilities arise on finest grid, which degrades accuracy
- MDG-ICE(q=2) isoparametric quadrilateral elements
  - Initially very accurate, but non-monotonic convergence (far from asymptotic regime?)

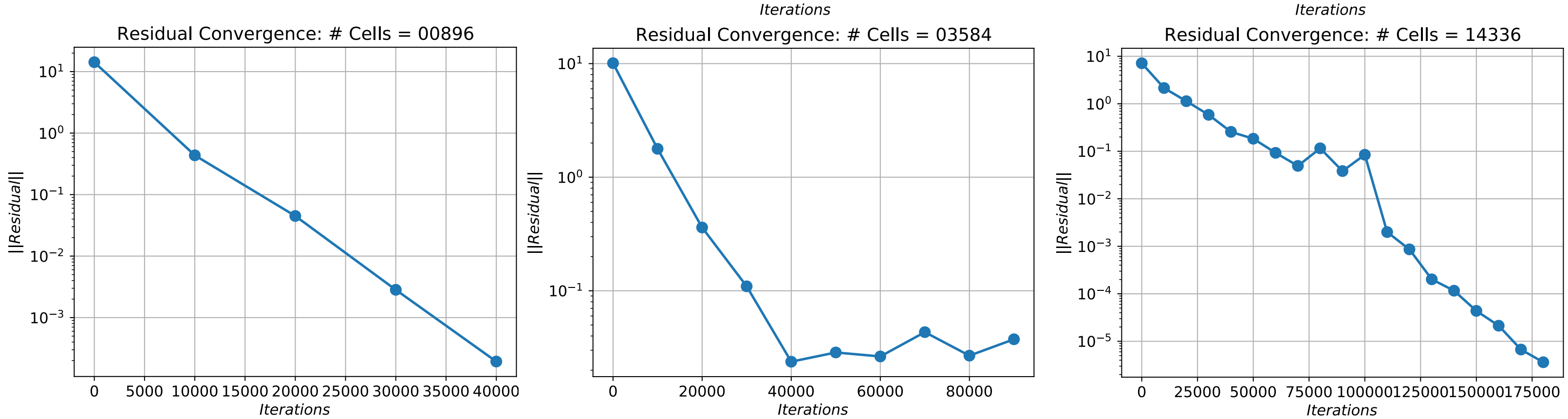


Bow Shock: DG Convergence History

DG(q=2) isoparametric quadrilateral elements with shock capturing

- Explicit time-stepping
- Excruciatingly slow convergence for steady flow.
- Converged until output quantities stopped changing beyond ~4 decimal places





Bow Shock: DG Work Units

DG(q=2) isoparametric quadrilateral elements with shock capturing

- Explicit time-stepping
- Excruciatingly slow convergence for steady flow.
- Converged until output quantities stopped changing beyond ~4 decimal places

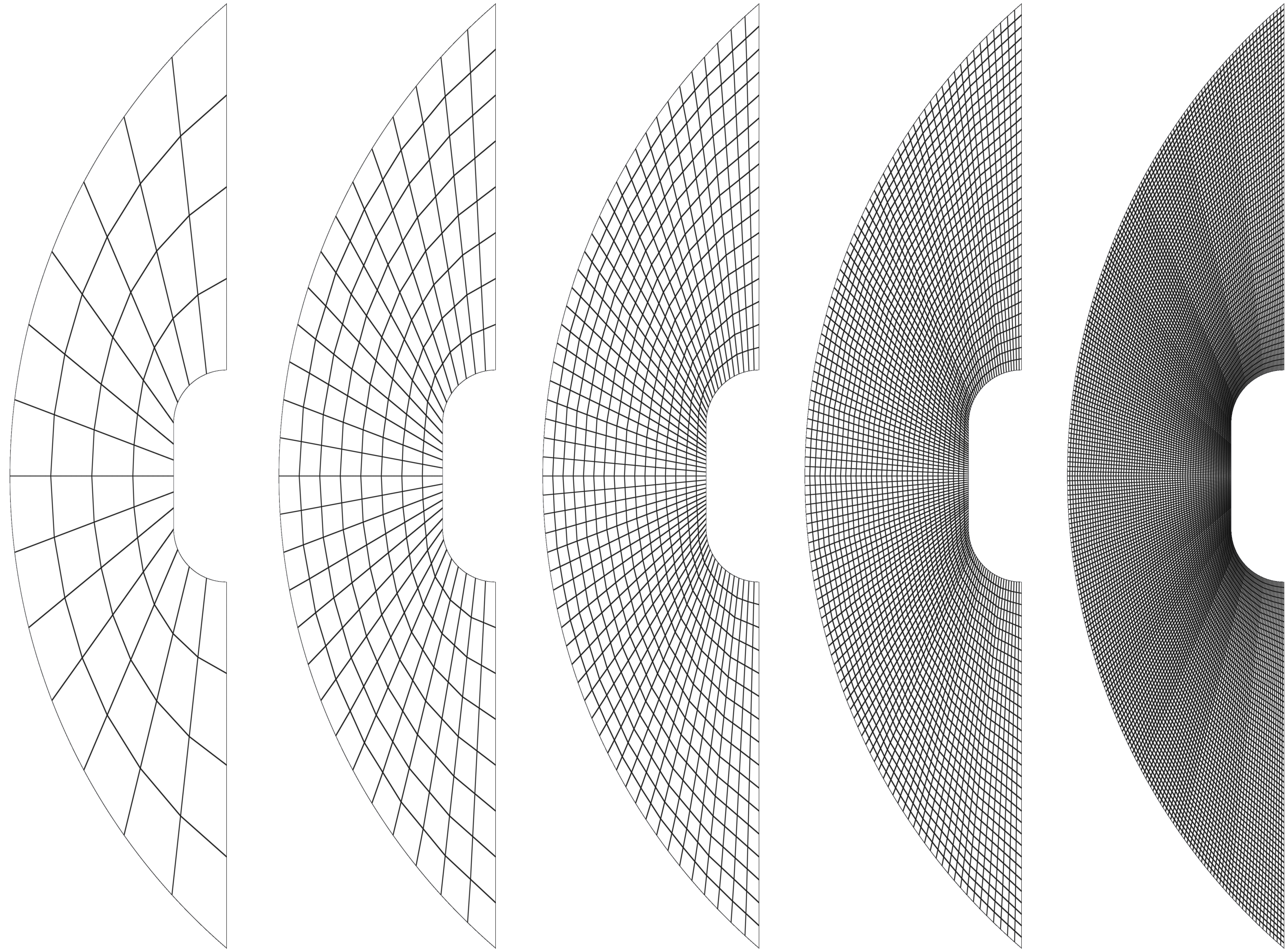
Work Units

- TauBench Score: 6.223
- Run on GPU:
  - Normalized also by GPU vs. 1-Core CPU speedup: 62.627x

- 56 cells: 711.7
- 224 cells: 1566.8
- 896 cells: 3404.5
- 3584 cells: 9552.9
- 14436 cells: 65611.0

Bow Shock: Grids

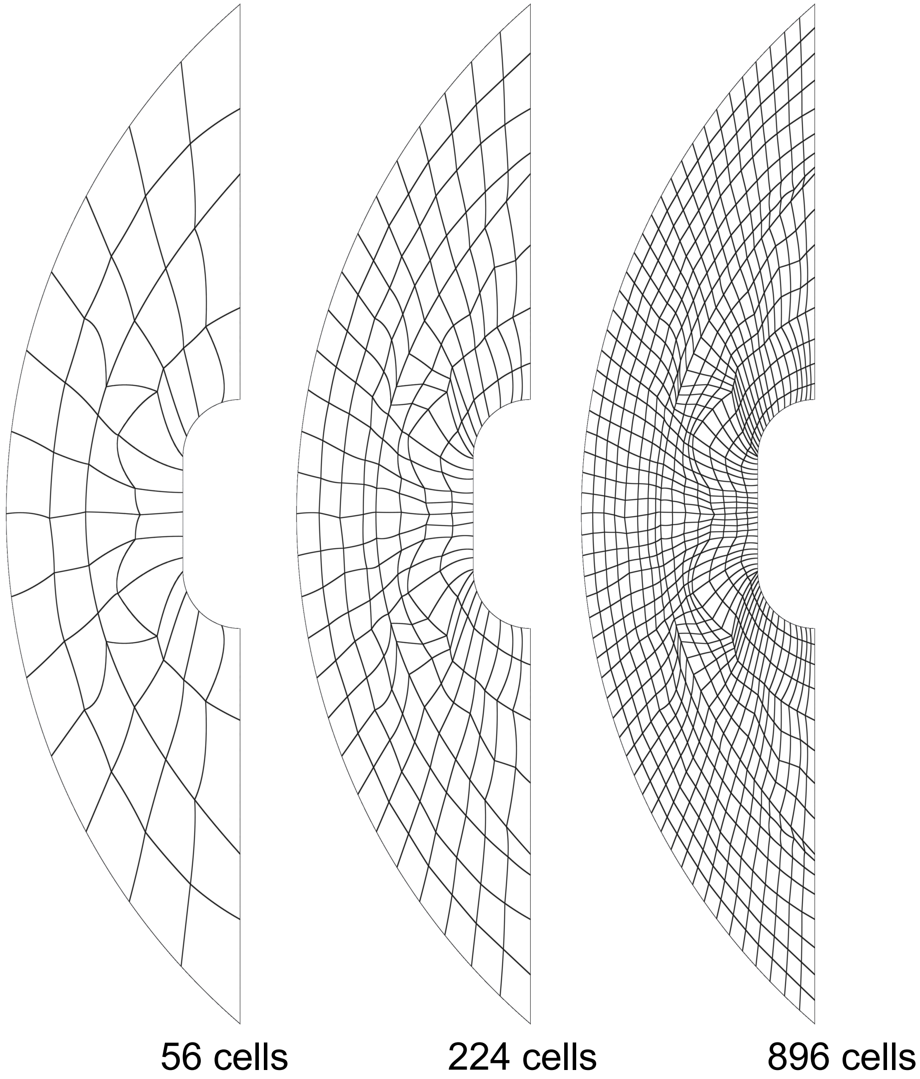
- DG(q=2) isoparametric quadrilateral elements with shock capturing
- Uniformly refined from initial grid
  - Projected geometry to analytic surfaces



56 cells 224 cells 896 cells 3584 cells 14336 cells

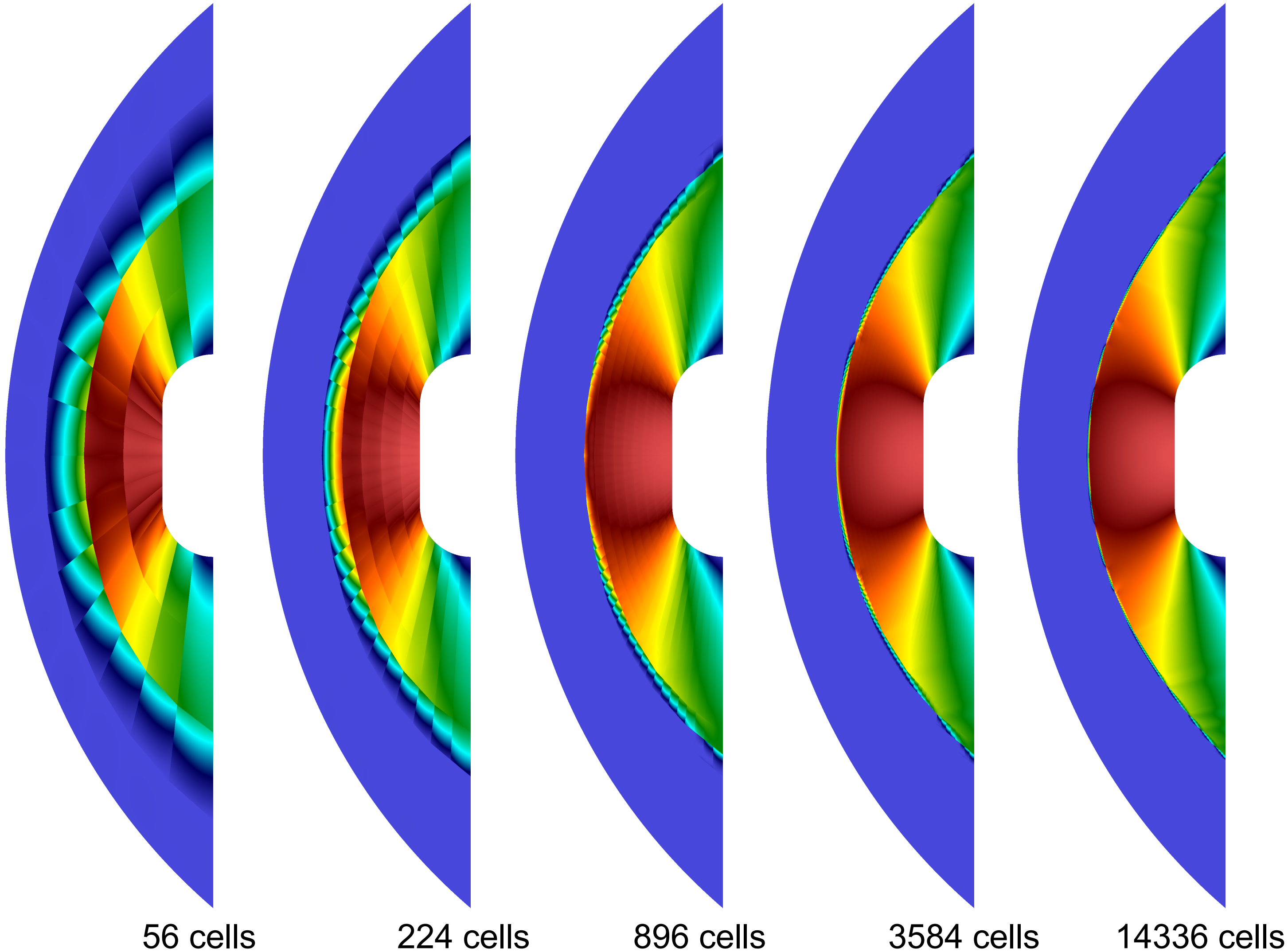
Bow Shock: Grids

- MDG-ICE(q=2) isoparametric quadrilateral elements
  - First solved for the initial fitted grid using 56 cells
  - Finer grid solutions were obtained by refining the coarse grid solution and restarting, so that the grid continues to move
- Points move along the boundaries, constrained by geometric boundary conditions.
  - At surface intersections, the points are fixed



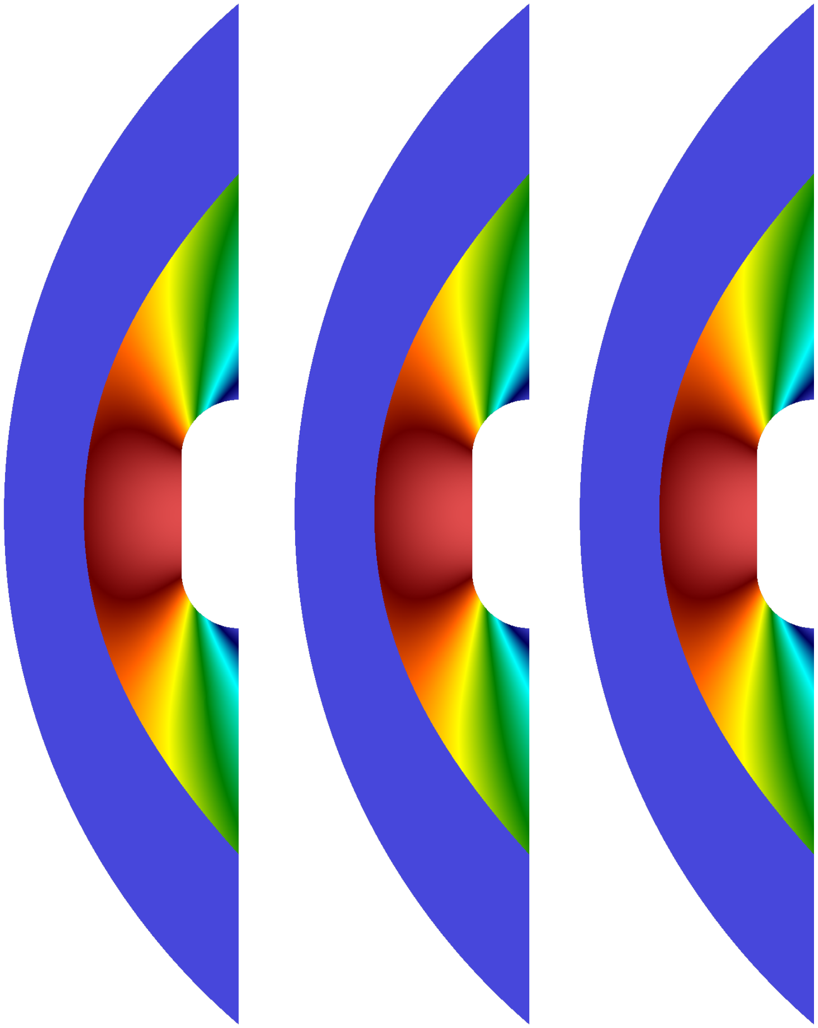
Bow Shock: Pressure

- DG(q=2) isoparametric quadrilateral elements with shock capturing
  - Plotted range (0,4)



Bow Shock: Pressure

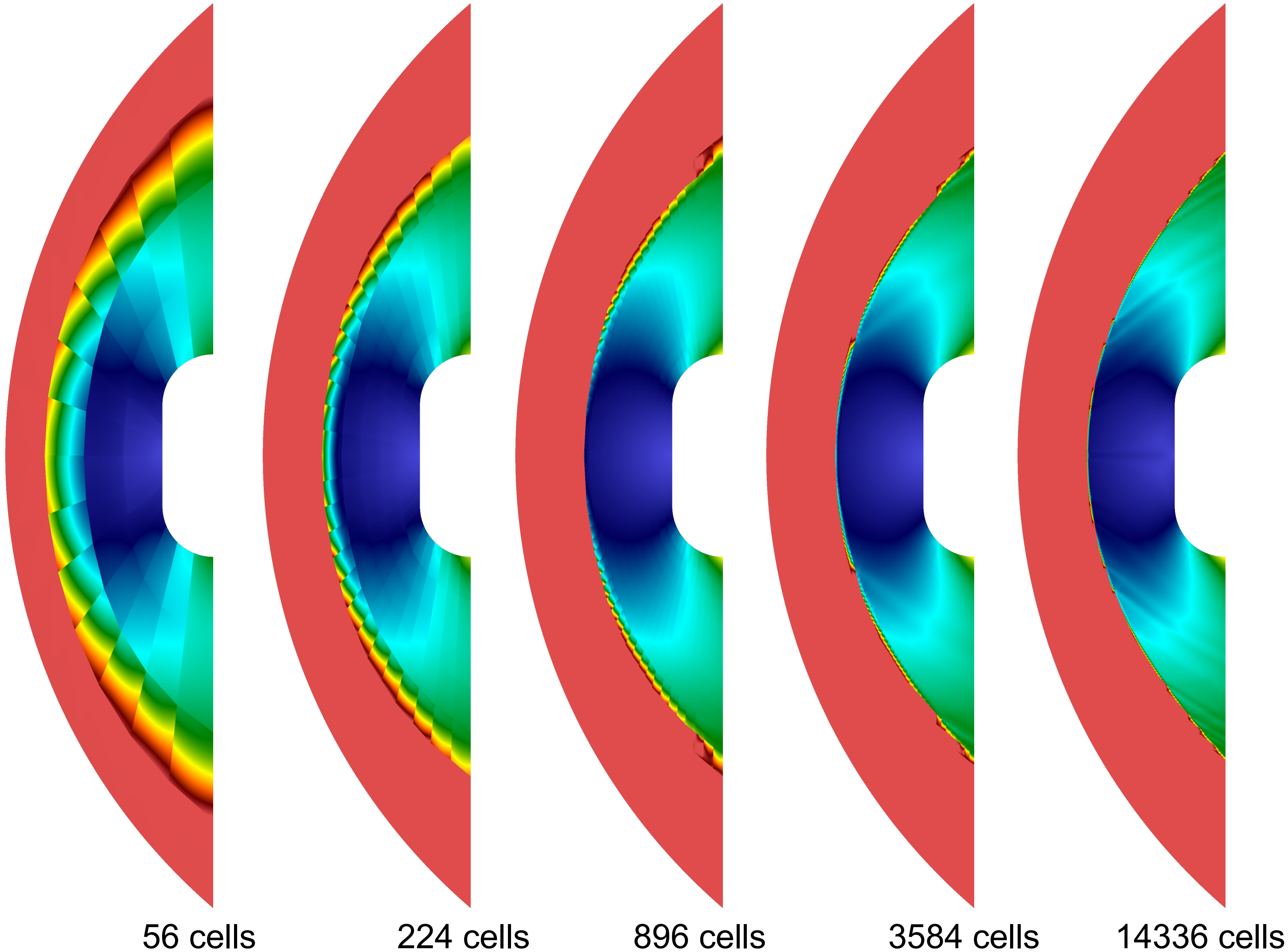
- MDG-ICE(q=2) isoparametric quadrilateral elements
  - Plotted range (1,21.068)



56 cells　224 cells　896 cells

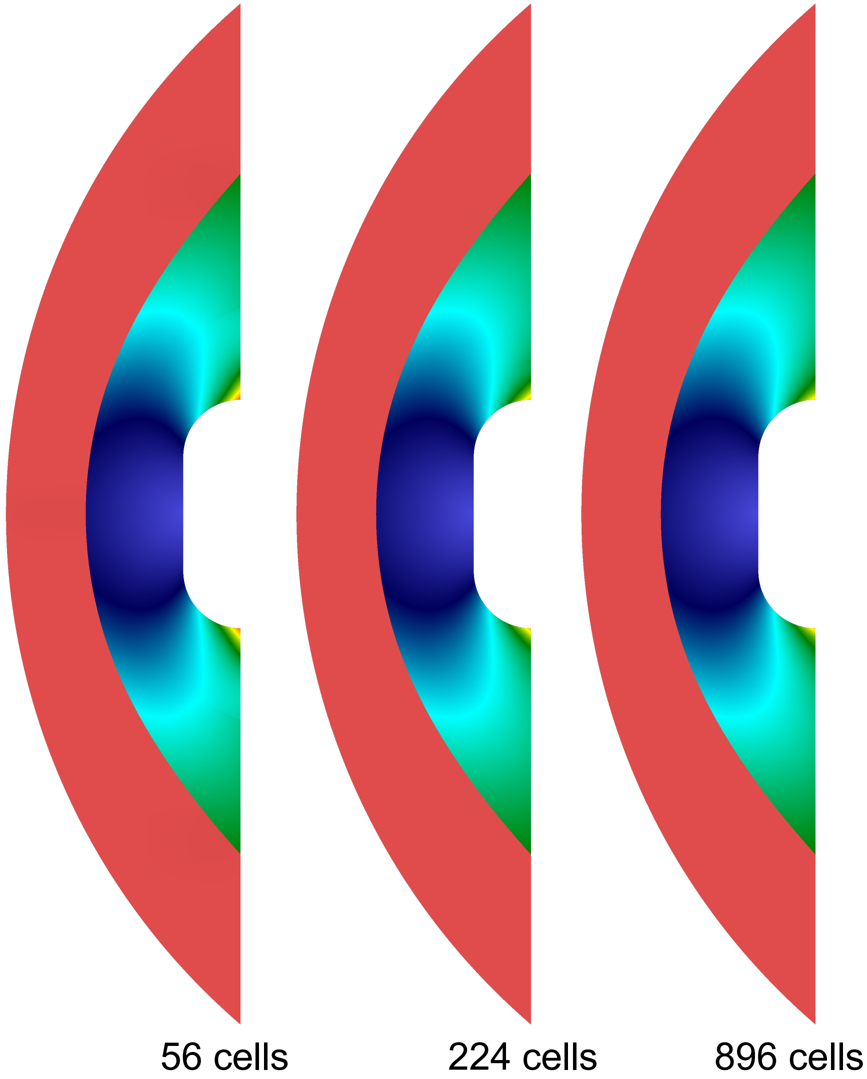
Bow Shock: Mach

- DG(q=2) isoparametric quadrilateral elements with shock capturing
  - Plotted range (0,4)



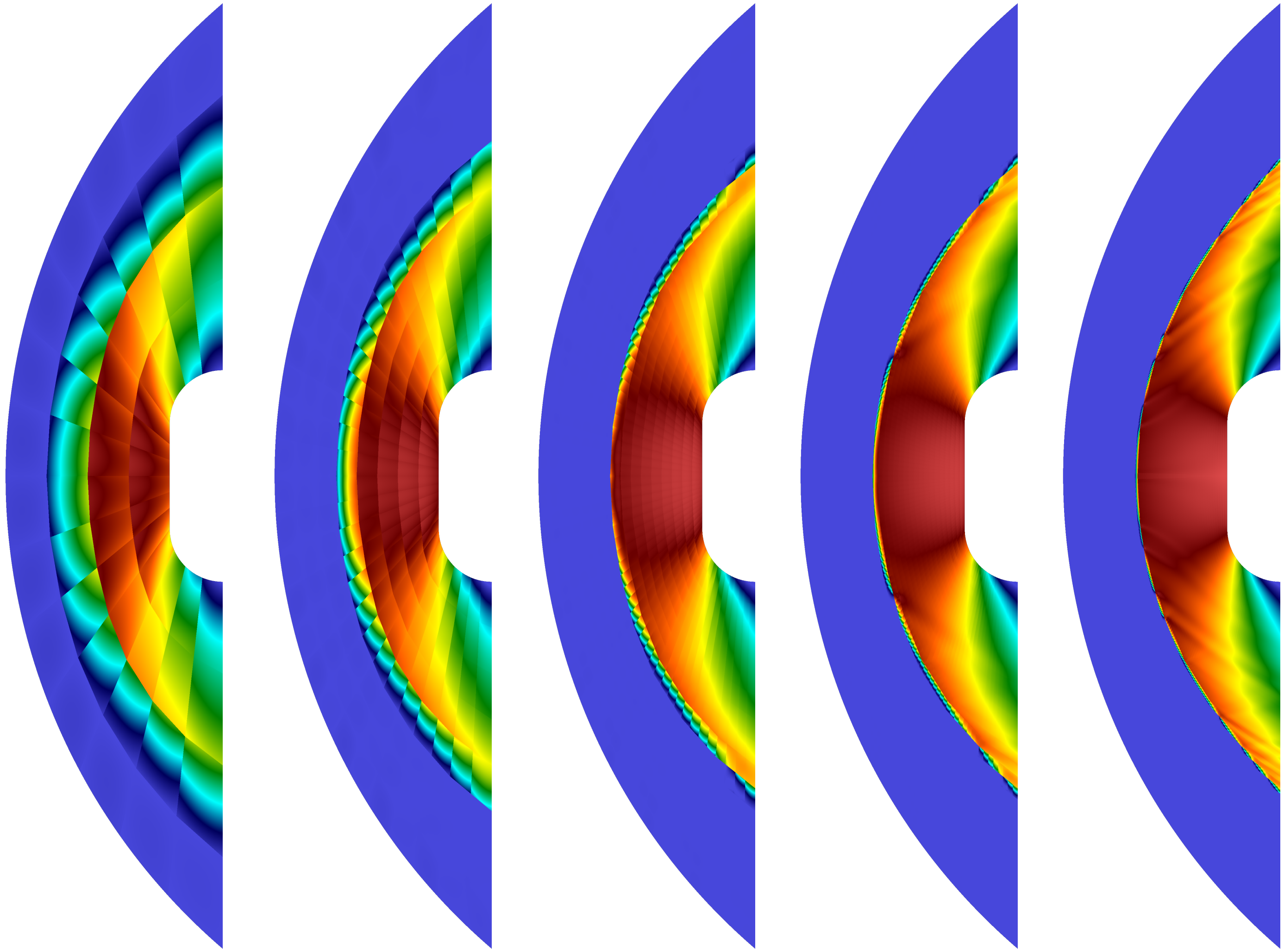
Bow Shock: Mach

- MDG-ICE(q=2) isoparametric quadrilateral elements
    - Plotted range (0,4)



Bow Shock: Density

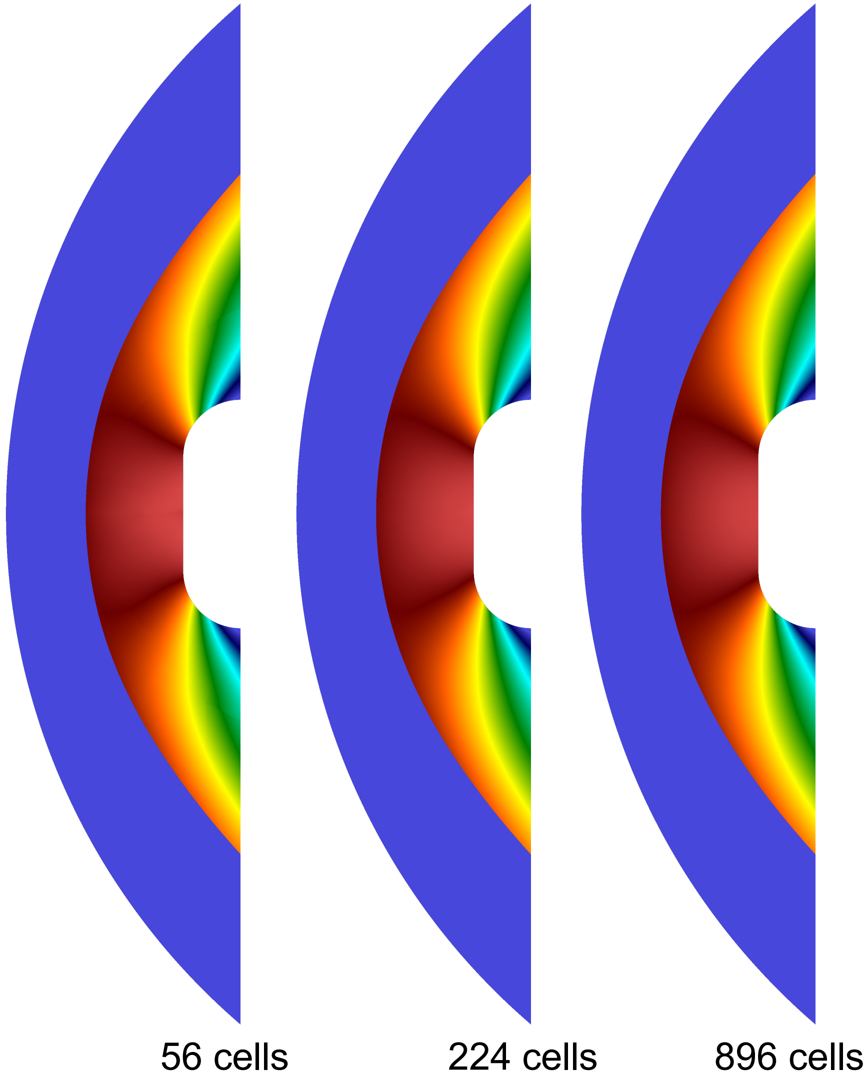
- DG(q=2) isoparametric quadrilateral elements with shock capturing
  - Plotted range (1,5.1)



56 cells | 224 cells | 896 cells | 3584 cells | 14336 cells

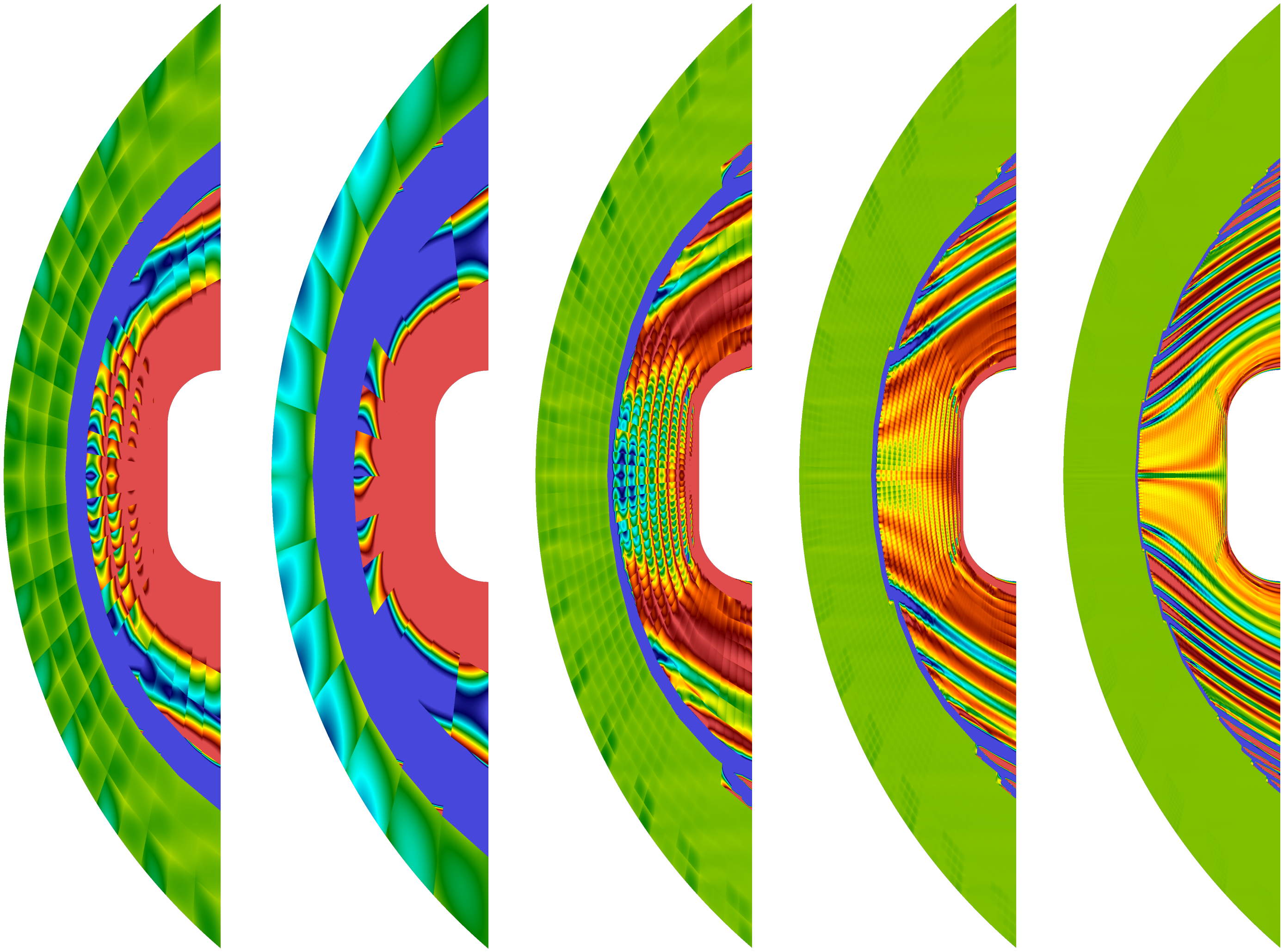
Bow Shock: Density

- MDG-ICE(q=2) isoparametric quadrilateral elements
  - Plotted range (1,5.1)



Bow Shock: Stagnation Enthalpy

- DG(q=2) isoparametric quadrilateral elements with shock capturing
  - Stagnation Enthalpy = 14.7
  - Plotted range 14.6,14.8



56 cells 224 cells 896 cells 3584 cells 14336 cells

Bow Shock: Stagnation Enthalpy

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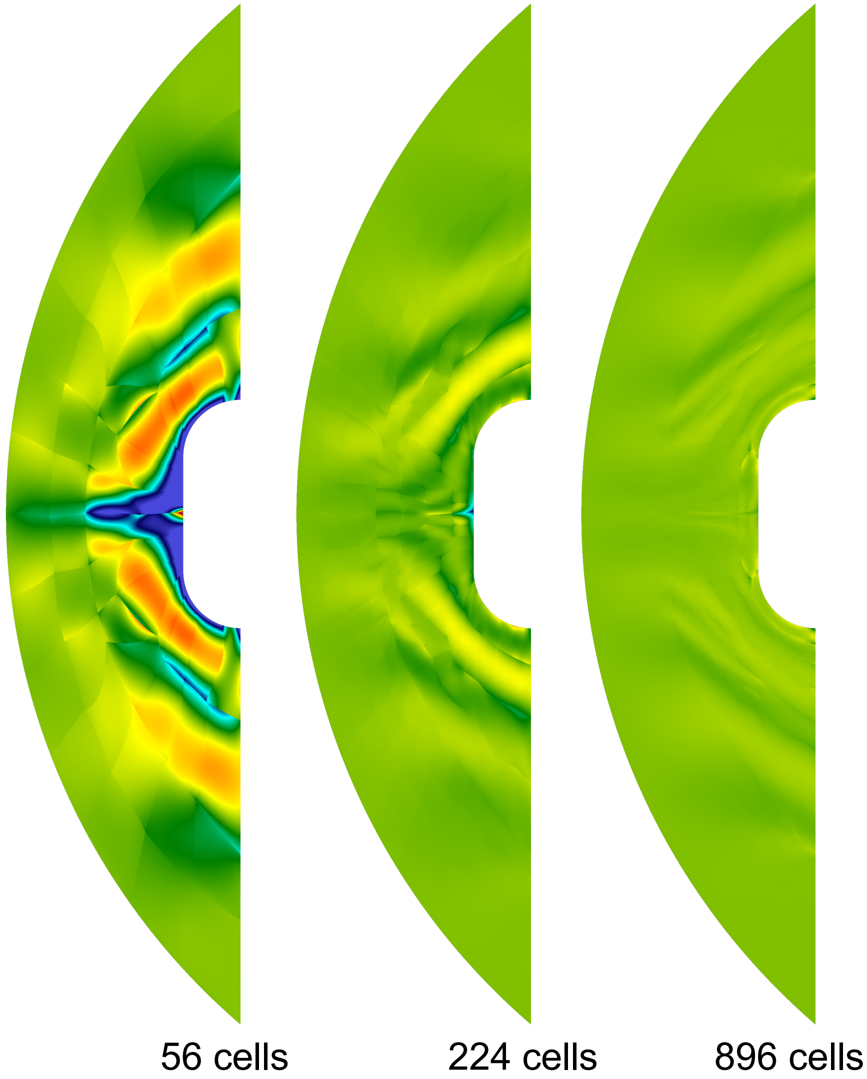


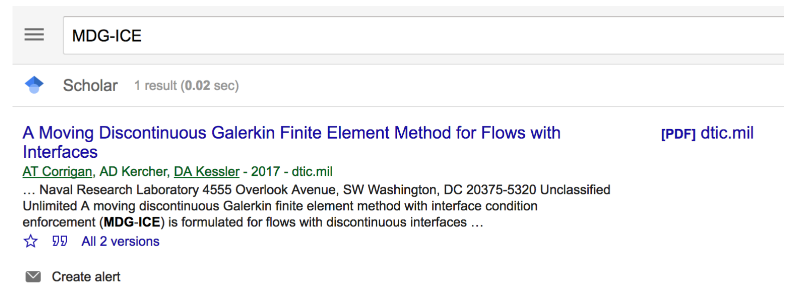
Conclusions

- **Were the verification cases helpful and which ones were used?**
  - The smooth bump test case helped verify the high-order accuracy of the DG method with curved geometry
- **What improvements are needed to the test case?**
  - Nested structured grids might give a better indication of the order of convergence
  - Fully unstructured grids would further stress:
    - The accuracy of shock capturing schemes
    - The robustness of shock fitting schemes.
- **Did the test case prompt you to improve your method/solver?**
  - Yes, we implemented a higher-order flux projection.
- **What worked well with your method/solver?**
  - MDG-ICE fits the curved bow shock with high accuracy on a coarse grid
- **What improvements are needed to your method/solver?**
  - A fast implicit solver: linear solver scalability and efficiency
  - Improving the nonlinear Levenberg-Marquardt regularization strategy to optimally balance robustness and efficiency

References

- NRL Memo Report / Submitted Manuscript:
  - A Moving Discontinuous Galerkin Finite Element Method for Flows with Interfaces
  - Google Scholar: **MDG-ICE**

  www.dtic.mil/docs/citations/AD1042881



- AIAA SciTech 2018 Papers
  - AIAA-2018-0136. Flow Characteristics of a Recirculating Flameless Combustor Configuration
  - AIAA-2018-1247. Jet Noise Simulation using a Higher-Order Discontinuous Galerkin Method
  - AIAA-2018-1252. Noise Characteristics of a Supersonic High Aspect Ratio Rectangular Nozzle for Ideally Expanded Jet Flow
  - AIAA-2018-1485. A Hybrid DSMC-discontinuous Galerkin finite element method for rarefied gas flows