CI2 – Inviscid Strong Vortex-Shock Wave Interaction

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Outline

- Discretization Scheme
- Verification Case → VI1:Inviscid convected Vortex
- Problem Description → CI2:Inviscid Strong Vortex-Shock Wave Interaction
- Computational Results
- Conclusions

Discretization Scheme

- Space Solver: FR/CPR
- Time Integration: RK3 method

Formulation (Conservation Laws):

$$\frac{\partial Q}{\partial t} + \vec{\nabla} \cdot \vec{F}(Q) = 0$$

$$\int_{v_i} \left(\frac{\partial Q}{\partial t} + \vec{\nabla} \cdot \vec{F}(Q)\right) W dV =$$

$$\int_{v_i} \frac{\partial Q}{\partial t} W \, dV + \int_{\partial V_i} W \, \vec{F}(Q) \cdot \vec{n} dS - \int_{v_i} \vec{\nabla} W \cdot \vec{F}(Q) dV = 0$$

Final Form

$$\frac{\partial Q_{i,j}}{\partial t} + \pi_j \left(\nabla \cdot \vec{F}(Q_i)\right) - \pi_j^{\nu} \left(\nabla \cdot \vec{F}^{\nu}(Q_i, \vec{R}_i)\right) + \frac{1}{|V_i|} \sum_{f \in \partial V_i} \sum_{l} \alpha_{j,f,l} \left([F^n]_{f,l} - [F^{\nu,n}]_{f,l}\right) S_f = 0$$

Smooth Limiter

General Form:

$$u_{i,fp} = \bar{u}_i + \tilde{\phi}_i(1,y)(u_{i,fp} - \bar{u}_i)$$

where $\tilde{\phi}_i(1,y) = \sigma_i + (1-\sigma_i)\,\phi_i(1,y)$

$$\phi_i(1,y) = \widetilde{min}(1,y) = \begin{cases} -\frac{4}{27}y^3 + y, & y < 1.5 \\ 1, & y \geq 1.5 \end{cases}$$

σ is determined by "smoothness indicator"

$$\sigma_i = \begin{cases} 1, & S_i < M - h \\ \frac{1}{2}\left(1 - \sin\left(\frac{\pi}{2}\frac{S_i - M}{h}\right)\right), & M - h < S_i < M + h, \\ 0, & S_i > M + h \end{cases}$$

Smooth Limiter

S_i is smoothness indicator, which is defined as

$$S_i = log_{10} \frac{\langle u - \hat{u}, u - \hat{u} \rangle_i}{\langle u, u \rangle_i}$$

$\hat{u}$ is one degree lower space's L2 projection of u.

M=-3.2 and h=1 are applied for all cases.

For more information about limiter and scheme please refer to:

Li, Yanan, and Zhi J. Wang. "A convergent and accuracy preserving limiter for the FR/CPR method." *55th AIAA Aerospace Sciences Meeting*. 2017.

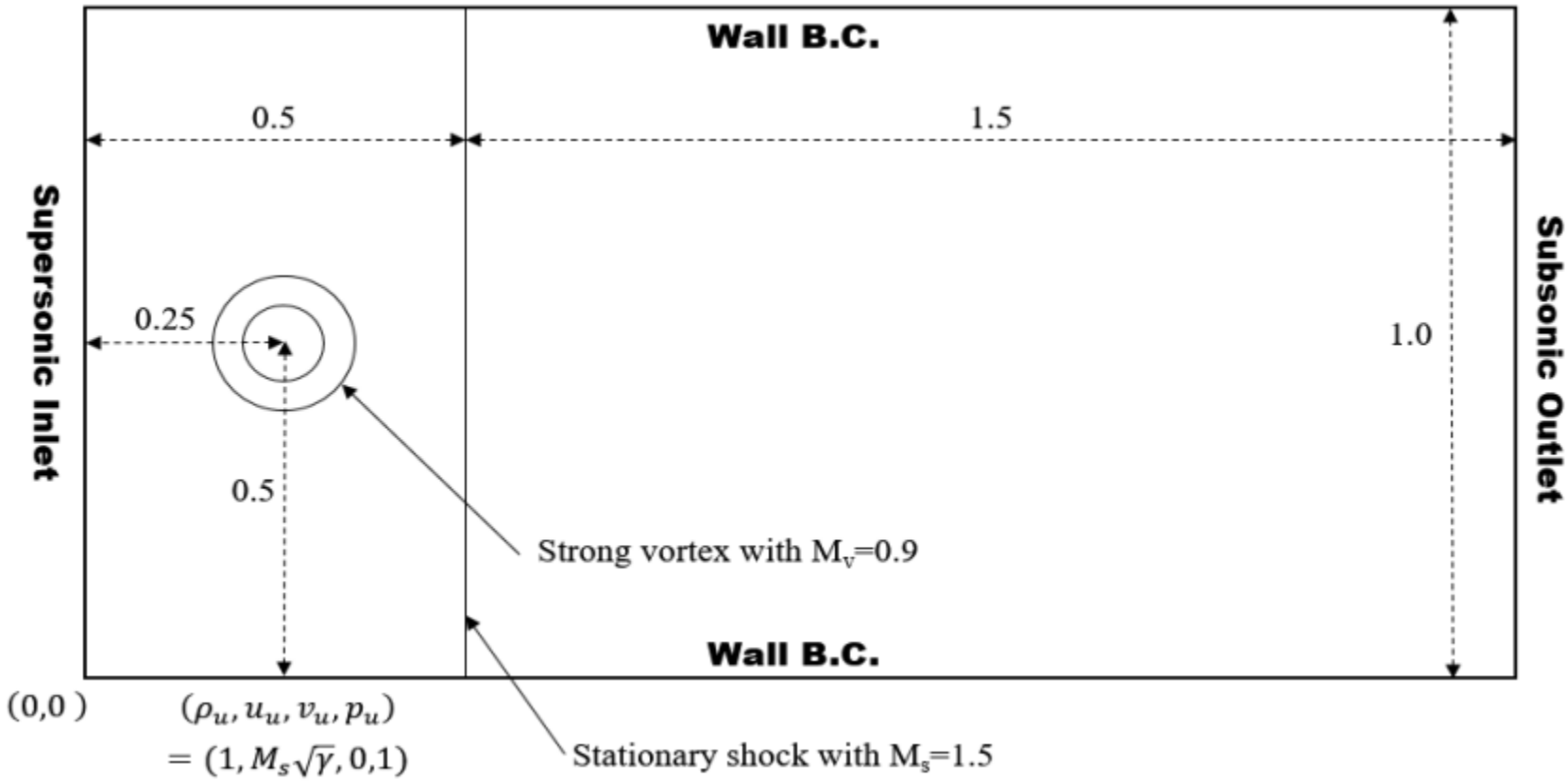
Yu, Meilin, Z. J. Wang, and Yen Liu. "On the accuracy and efficiency of discontinuous Galerkin, spectral difference and correction procedure via reconstruction methods." *Journal of Computational Physics* 259 (2014): 70-95.

Verification : Vl1-Inviscid Converted Vortex

P2(T=50) limiter					
N_cell	h	u_L2_error	v_L2_error	order_u	order_v
256	0.020833333	2.8069144723934900	2.8079207977044500		
1024	0.010416667	1.5102071773645600	1.4719045626825000	0.89	0.93
4096	0.005208333	0.0894953965449293	0.0726530489601594	4.08	4.34
16384	0.002604167	0.0021178532255229	0.0016970421683930	5.40	5.42
P3(T=50) limiter					
N_cell	mesh_size	u_L2_residual	v_L2_residual	order_u	order_v
256	0.0156250000	2.1510329168290600	2.7499337777306400		
1024	0.0078125000	0.1080328621037420	0.1015299838558880	4.32	4.76
4096	0.0039062500	0.0010576846176196	0.0008869180482861	6.67	6.84
16384	0.0019531250	0.0000416527019559	0.0000289939246621	4.67	4.93

P2(T=50) No limiter					
N_cell	h	u_L2_error	v_L2_error	order_u	order_v
256	0.020833333	2.8069144723934900	2.8079207977044500		
1024	0.010416667	1.5102071773645600	1.4719045626825000	0.89	0.93
4096	0.005208333	0.0894953965449293	0.0726530489601594	4.08	4.34
16384	0.002604167	0.0021178532255229	0.0016970421683930	5.40	5.42
P3(T=50) No limiter					
N_cell	mesh_size	u_L2_residual	v_L2_residual	order_u	order_v
256	0.0156250000	2.1727416920924200	2.7635931154149700		
1024	0.0078125000	0.1080328620934570	0.1015299838459960	4.33	4.77
4096	0.0039062500	0.0010576846078093	0.0008869180388659	6.67	6.84
16384	0.0019531250	0.0000416526926961	0.0000289939174558	4.67	4.93

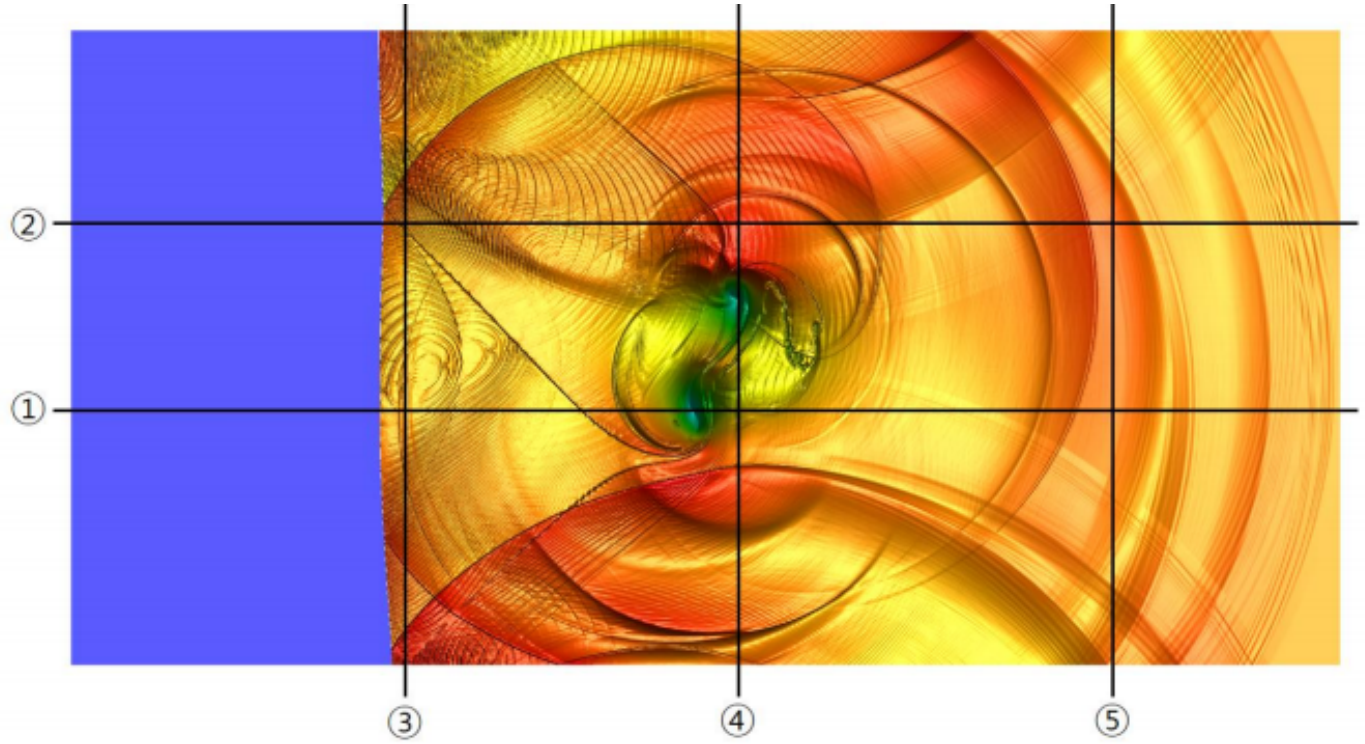
Problem Description



NUMERICAL BOUNDARY

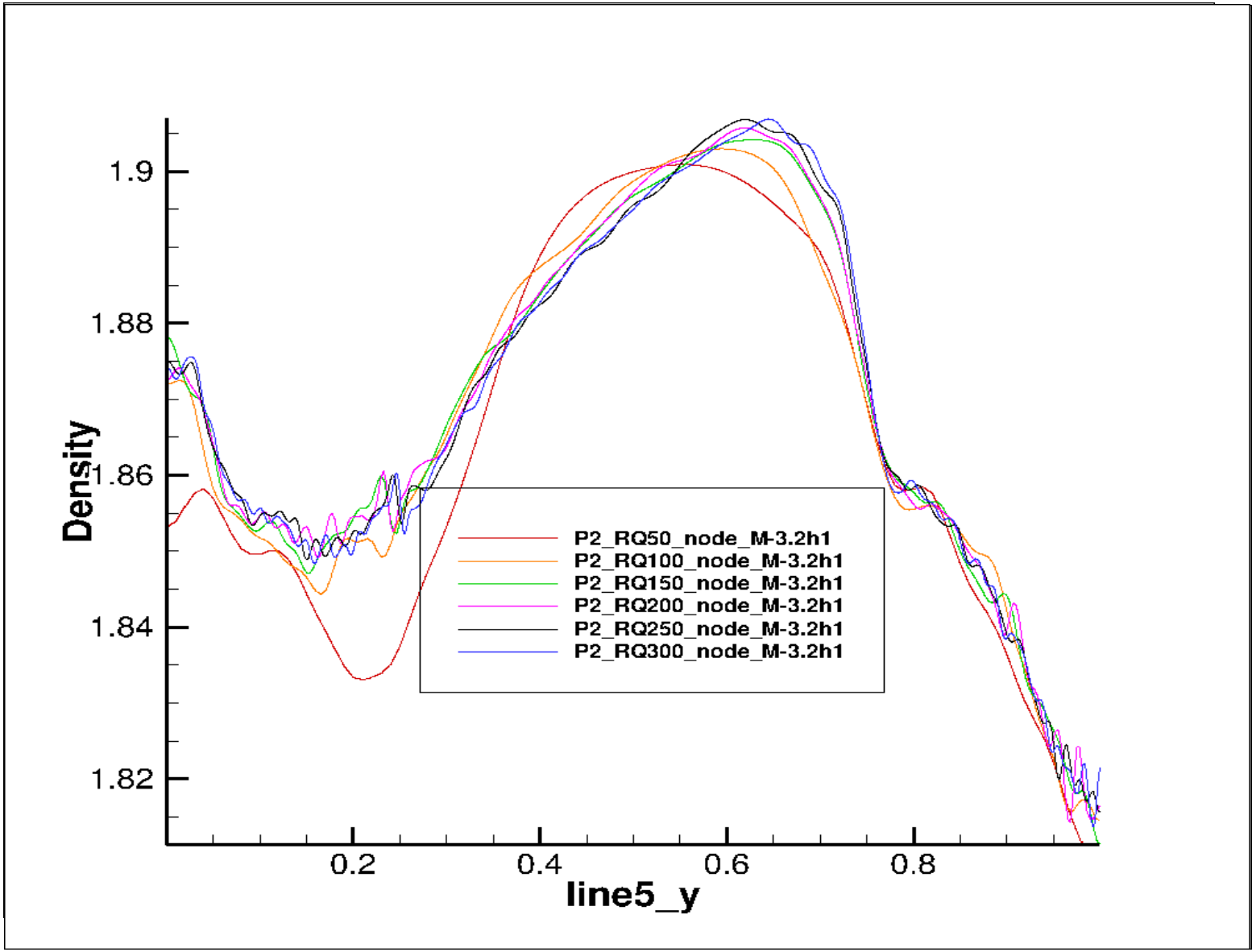
Inlet: Fixed all Outlet: Fixed all Wall: Slipped Wall

Computational Results

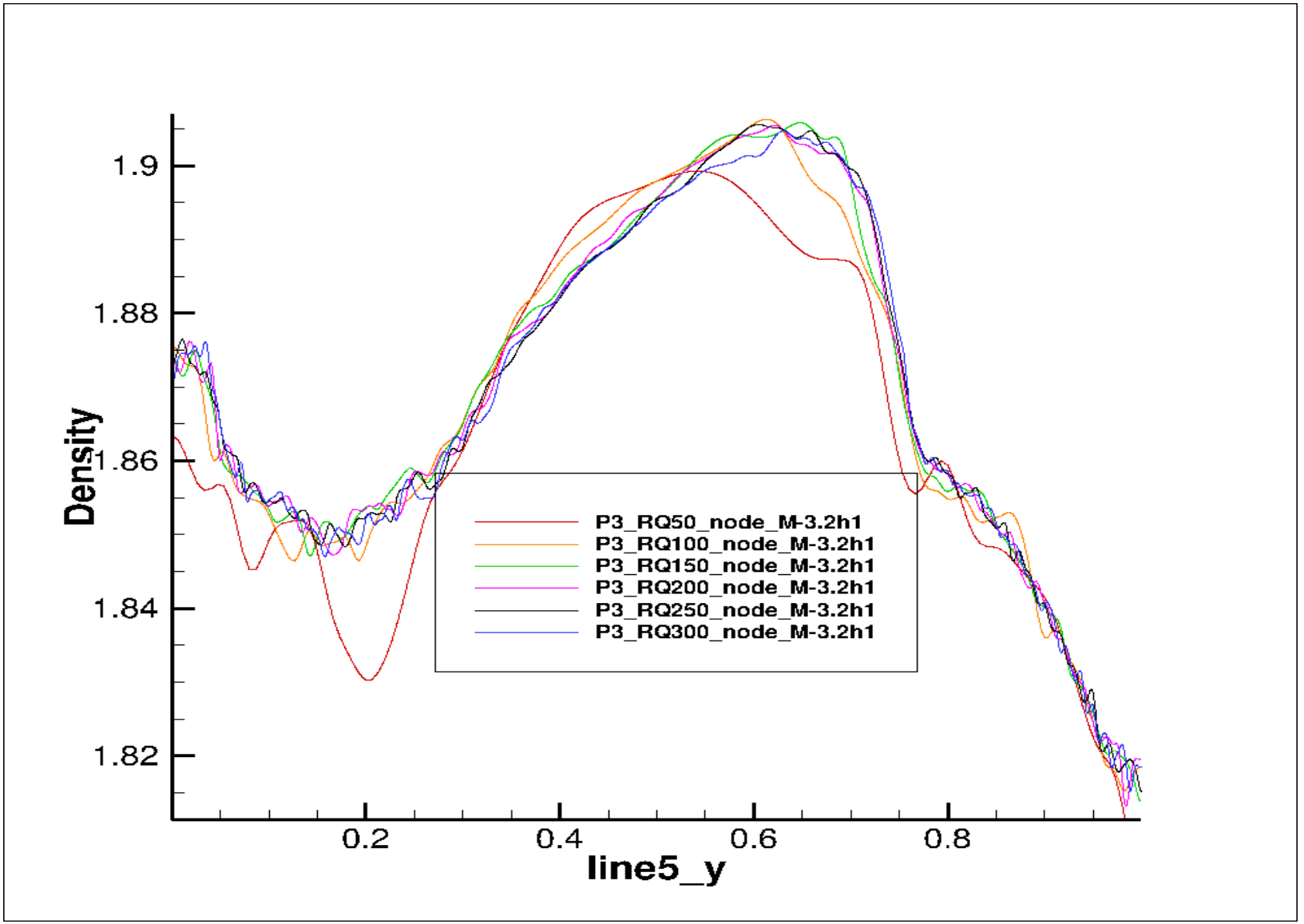


- Five selected lines' density distribution to evaluate solutions;
- Schlieren contours which cover entire domain and vortex center at t=0.7s, respectively.

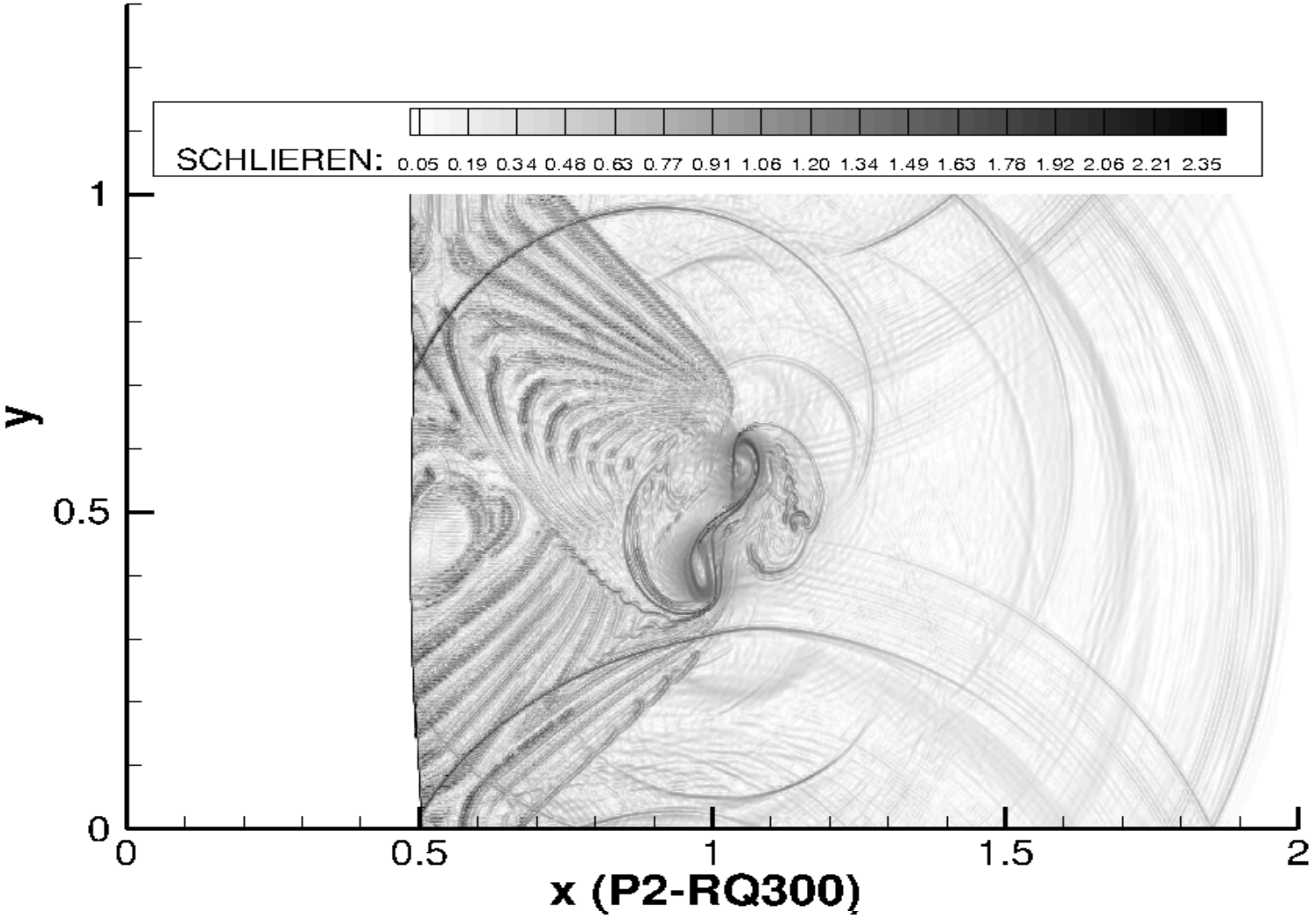
Selected Lines-P2



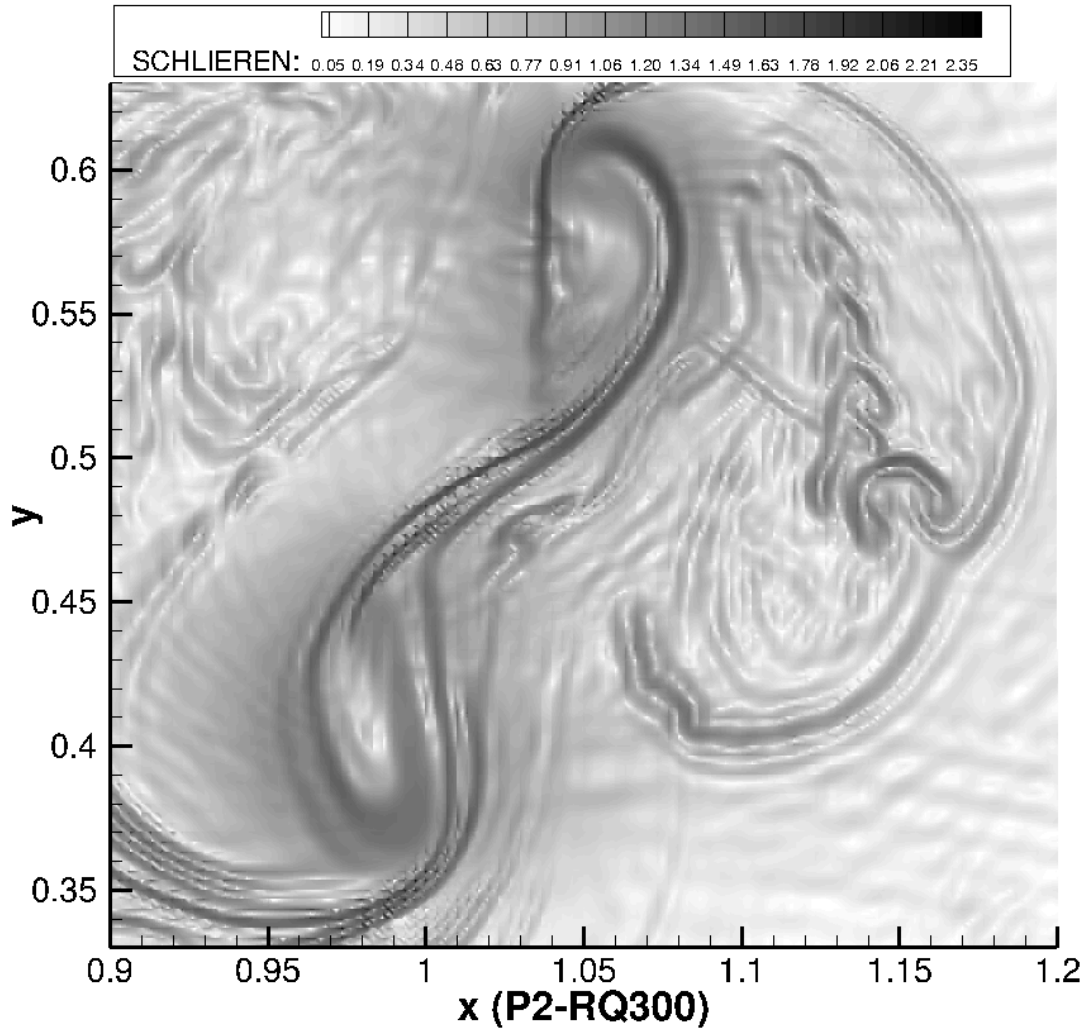
Selected Lines-P3



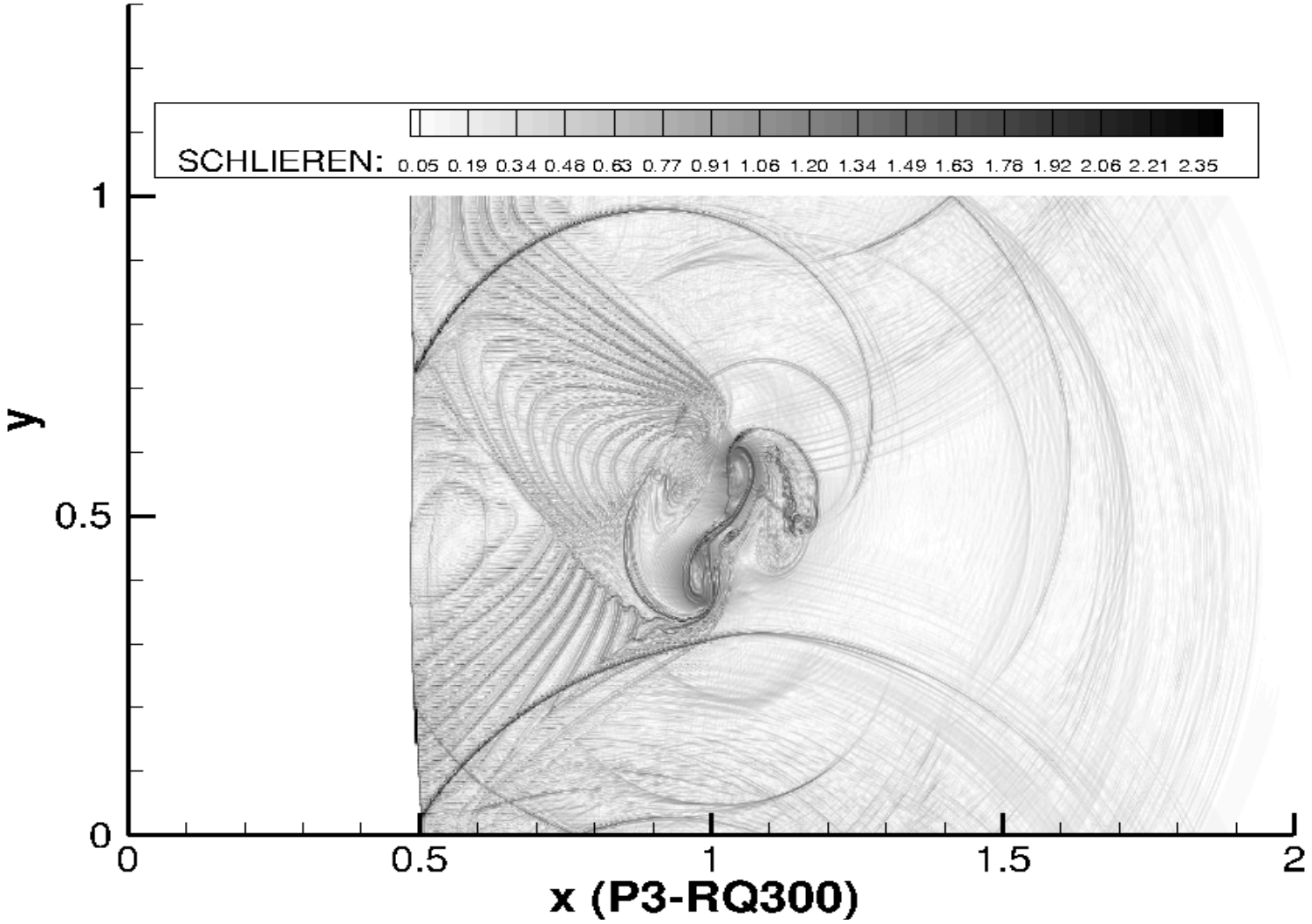
Schlieren Contours-P2



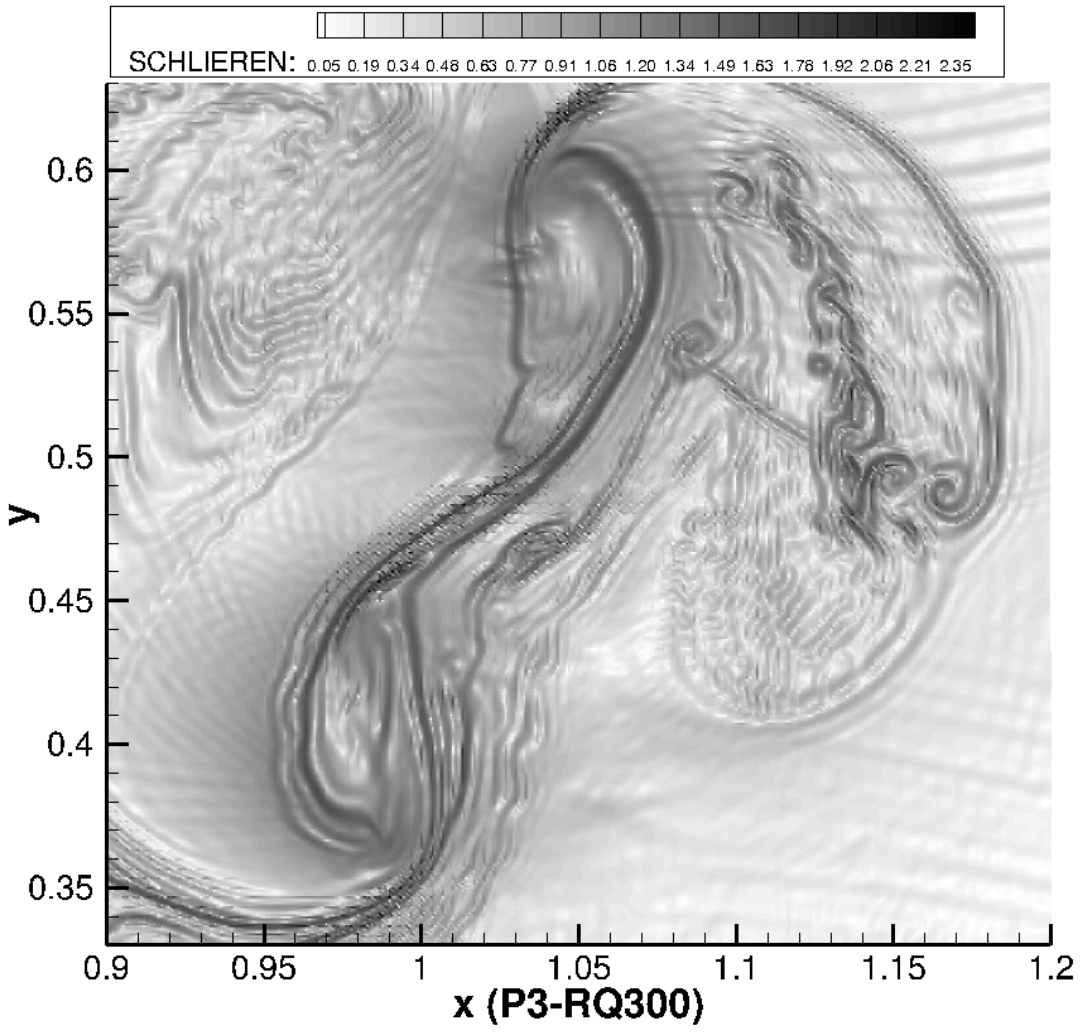
Schlieren Contours-P2



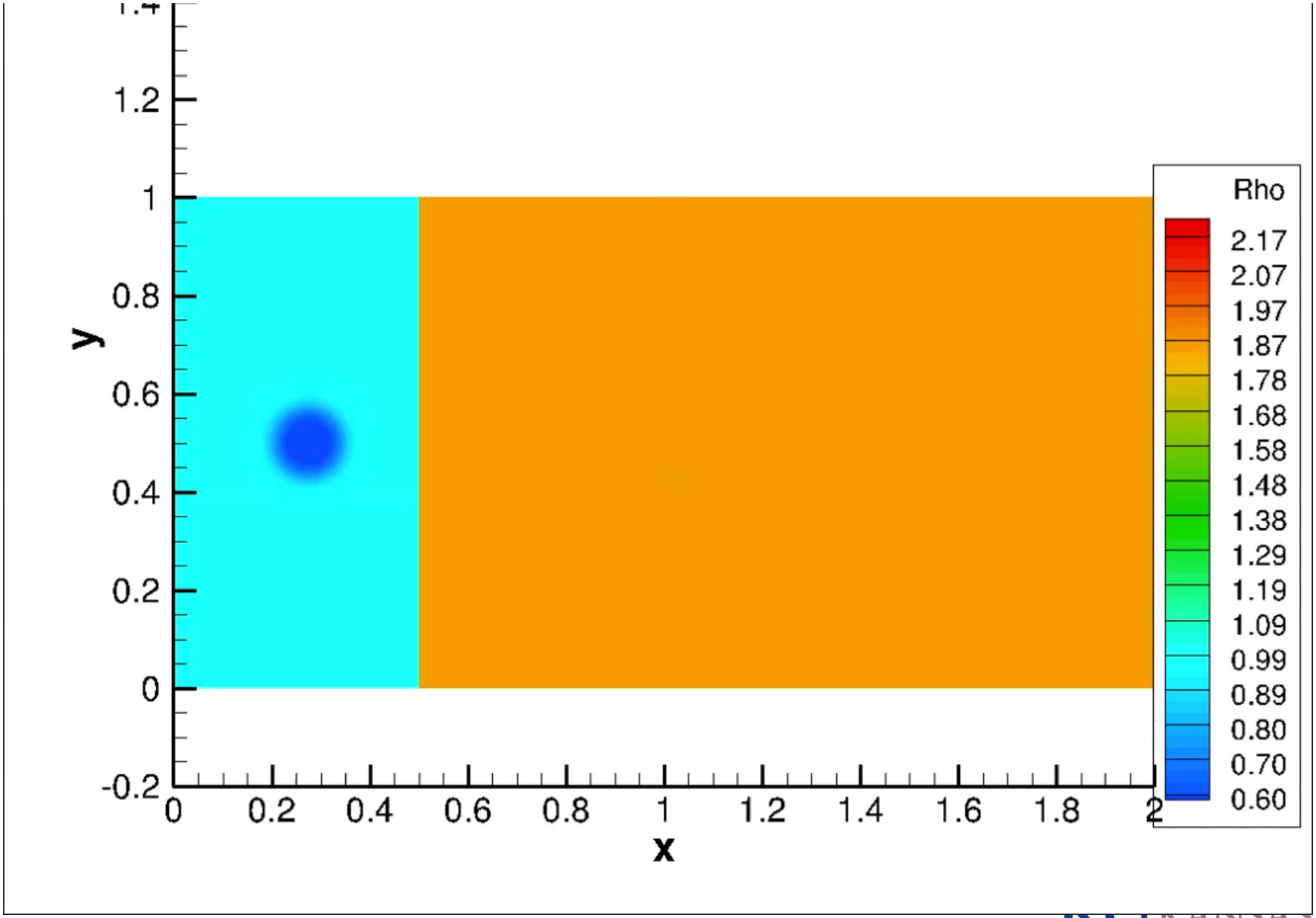
Schlieren Contours-P3



Schlieren Contours-P3



Movie – P2 (RQ800)



Conclusions

- Smooth Limiter for CPR/FR has the ability to preserve high-order accuracy in smooth region;
- Whether high frequency oscillations behind the shock are physical or not are still uncertain;
- CPR/FR could capture detailed flow structures while suppressing pseudo oscillations near strong shock with proper choice of limiter parameters;
- We improved our limiter with this case
- We hope to remove user tunable parameters.

Cost

vortex_shock

Category	N_cell	h	work_unit	machine	cpu_time
P2_RQ50	5000	0.0047140	9.77	gpus01	9.52E+01
P2_RQ100	20000	0.0023570	86.55	gpus01	8.43E+02
P2_RQ150	45000	0.0015713	297.23	gpus01	2.90E+03
P2_RQ200	80000	0.0011785	724.39	gpus01	7.06E+03
P2_RQ250	125000	0.0009428	1290.41	gpus01	1.26E+04
P2_RQ300	180000	0.0007857	2299.95	gpus01	2.24E+04

Category	N_cell	h	work_unit	machine	cpu_time
P3_RQ50	5000	0.0035355	27.26	gpus01	2.66E+02
P3_RQ100	20000	0.0017678	237.31	gpus01	2.31E+03
P3_RQ150	45000	0.0011785	779.98	gpus01	7.60E+03
P3_RQ200	80000	0.0008839	1901.28	gpus01	1.85E+04
P3_RQ250	125000	0.0007071	3589.23	gpus01	3.50E+04
P3_RQ300	180000	0.0005893	6218.68	gpus01	6.06E+04