5th Workshop in High-Order Methods (HiOCFD5)

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January 6, 2018

Throughout these notes, I use the term EOC for Empirical Order of Convergence and AMR for Adaptive Mesh Refinement (for any dynamic mesh adaptation in h).

1 Farshad Navah

- Slides 1-4: He identified **verification** with **"cumulative evidence"** of convergence to governing system and distinguished it from validation.
- Slides 5-7: He singled out RANS for notorious grid-dependence. When we have things like multiple surfaces with different boundary conditions, we can only eliminate so many parts of a problem. He called this floor below which we cannot simplify further "irreducible high complexity."

1.1 Method of Manufactured Solutions (MMS) framework

- Slide 8: Definition of "arbitrary PDE."
  1. Make up a function you like that has some features to which you can impute a physical character or significance.
  2. Add a forcing function or otherwise modify your governing system of equations into a form whose solution is the function you made up.
- Aside
  - Q: Should the forcing be integrated analytically or via quadrature?
  - A: We just use quadrature and evaluate it at the quadrature nodes.
- Elimination and simplification to decouple things like BC from things like turbulence models.
- Slide 10: Solution Adaptive Numerical Simulator (SANS). I don't dispute the footnotes, but the effect of dealiasing[4] can vary significantly at different resolutions (in h and p) for rational flux functions.
- Slide 13: **Negative SA**. McGill's RANS codes use a version of Spalart-Almaras with *negative* Prandtl-number formulae that allow negative eddy viscosity without destructive Gibbs. Galbraith said throughout the workshop that this is the *only* turbulence model suitable for high-order RANS.
- Slide 22: Enables deep verification with minimal changes to existing code. Sensitivity analysis of model terms. Sensitivity to small perturbations in not-so-physically-meaningful places may correspond to sensitivity to literal bugs in code.
- Introduced compensated plots of error intended to "plateau" at theoretical order of accuracy. This should be more widespread than it is. Logarithmic plots are easy to cheat.

Rule of thumb: **Three resolutions** with the same empirical order of accuracy are necessary to "pass" verification with respect to both exact and manufactured solutions. Unchallenging/ "easy" problems can hide numerical and model errors (e.g. insufficient diffusion). Order of accuracy can also vary slightly from conserved variable to conserved variable. Convergence in drag hides problems. Drag is easy from this point of view. Small errors or modest disagreement in drag can be large in global L_p.

- Slides 30-31: Isoparametric mapping can produce surface normals at boundaries that, with reflecting BC, can impair convergence. Convergence can be recovered with exact normals, if they are available.
- Slides 69-73: References.

Sometimes he had coarse cases (MMS RANS with SANS adjoint consistent) vary wildly in empirical order of accuracy, but recover it at fine meshes.

2 CI2: Inviscid strong vortex-shock wave interaction.

The use of cell averages for error samples follows Woodward & Colella (1984) *JCP* for convergence analysis in flows with shock waves, but using such averages in the L_2 norm instead of the L_1 norm does not.

I missed most of Chongam Kim's talk.

2.1 Hojun You

- Slide 2: Seoul National University (SNU) codes 1 (hMLP) and 2 (hLMP_BD)
- Slide 6: At $P = 2$, simulations using the local Lax-Friedrichs flux leave a spurious ghost of the stationary shock that the vortex carries all the way back. Simulations with the Roe flux do not have this!
- Slide 10: Coarse meshes reproduce large wave trajectories, but the vortex does not split at $1/h \leqslant 100$.
- Slide 12: Nonmonotonicity was blamed on "inconsistency." I am sorry to have missed further context of this remark.

Errors at shocks transport downstream and accumulate.

2.1.1 Sampling along Line 1

Mostly slide 13.

- Converges slower than first-order on coarse meshes, and better than first-order on fine meshes.
- L_2 norm for error at $P = 2$ was usually lower than L_2 error at $P = 3$.
- Mesh type matterd about as much as polynomial order.
- Perhaps a smoother IC would help? I agree. Majda & Osher (1977) *Comm. Pure Appl. Math.* said so.

2.1.2 Lines 2 through 5

- Slide 14: SNU1 had lower L_2 error in putatively smooth regions than SNU2, yet much higher total variation except on M250.
- Slide 17: Line 5: First-order convergence of hMLP_BD (and, at very fine meshes, mHLP as well). Such insensitivity to mesh, scheme, and polynomial order is expected for capturing irregular waves, such as unsteady fan heads and tails, across element faces. I suspect such "kinks" overwhelmed other errors.

2.2 Philip Johnson

Student of Eric Johnsen. Invented compact gradient recovery (CGR), a DG numerical flux for diffusive terms that replaces the second scheme of Bassi & Rebay's (BR2) face averages with recovered solution (via interface-centered binary (ICB) recovery here). Higher-resolution than BR2, and allows higher diffusion number (6 times larger at $P = 3$) for explicit time marching purposes.

Van-Leer's (2005) recovery concept is central to all of their work. Slide 2:

- One quadrature point per basis function.
- The SLAU2 Riemann solver is a descendent of AUSM.
- Discontinuity detector is based on element averages.
- ICB: Interface centered binary reconstruction. Cartesian-only for now, although the rest of the solver is fine for simplices.
- C-method of Reisner (PDE-based artificial viscosity $\kappa \sim C$) applied along bith coordinate directions (C_x and C_y, both obeying their own diffusion-reaction equations).

ICB illustrated slides 4-5.
Cases:

- Slide 6, VI1 smooth vortex: $2p + 2$ Convergence. See cited works for explanation.
- Slide 7, CI2 shock-vortex interaction: Outflow corruption only at $P = 3$. Unlike Seoul National University codes, Roe solver on triangles suffers ghost shock in their DG code.
- Slide 8, WS1 Taylor Green: ICB+CGR gets twice the peak kinetic energy dissipation rate (KEDR) in less than 1/3 the CPU time of conventional DG at $P = 2$ on 21.

2.3 Qilin Li

Slide 5: Measurement of energy fraction in highest Legendre mode as smoothness indicator. Like in Perrson & Peraire (2005) and Klöckner, Hesthaven & Warburton (2011). Unfiltered line plots of shock-vortex are oscillatory. Unknown if physical. No way to find out? Picture-norm self-convergence (termed "eyeball-norm" in this community). Their vortex bifurcation resembles Kim's as well as the reference solution. Kelvin-Helmholz roll-up resembles Kim's as well.

2.4 back to Chongam Kim

- VI1 inviscid vortex: DG doesn't converge well at 50 periods of advection on inviscid vortex VI1 without reconstruction.
- More effort to determine which oscillations were physical and which were not. He remarked about sensitivity to grid shape.

Composite vortex = vortex sitting behind shock without interacting directly. Very useful simplification to serve as a building block of CI2.

- Slide 31: Post-shock oscillations go downstream and impair convergence, especially when shock is not initialized or intended to be located at an interface in the mesh. Blamed on numerical flux.
- Curiously, they demonstrate worse error when the stationary shock is placed initially on a mesh interface than those cases in which the shock is initially placed within an element.

Projection error = error arising from representing even the initial conditions in the approximation space. Again, Majda & Osher (1977) could be a good theoretical starting point?

3 Discussion

- Ryan Glasby: No studies of temporal accuracy.
- RK8 is suspected to be responsible for Philip's much lower error for IV1 than everyone else's, which were done with TVDRK3 and SSPRK4.
- SLAU2 and Roe flux suffered the same stationary-shock artifact in Philip's code. Others did not suffer it with Roe.
- Kim is skeptical that smoothing ICs will get rid of this particular stationary-shock ghost artifact.
- Kim says CFL=0.9 too. Which makes temporal error is kind of a big deal, but still not same order of magnitude as spatial error. Travis Fisher agreed that spatial error overwhelms temporal in most high-order methods.

4 CI1: Bow shock

Mach 4 steady flow over a prescribed rounded-rectangle blunt body. Scott Murman's case. It is difficult for unsteady solvers (they "can barely run") to converge unsteadiness to machine precision. Murman's solver won't do it at $P = 7$.

4.1 Ben Couchman (presented by Marshall Galbraith)

SANS Contributions to Case CI1.

- Slide 3: Barter & Darmofal PDE artificial viscosity.
- Slide 4: Artificial viscosity pollutes the Gaussian bump, but only on coarse meshes.

Slide 6: L_2 norms of stagnation enthalpy error. Gaussian convolution of pressure error changed error in such a way that EOC went from 1 to 2 and more strongly favored low-order $P = 1$ except at high work units.
Also, Q = polynomial order of the representation of mesh faces (geometry).

- Slide 7: Same initial grid for all cases. Fully converged solutions on 900 meshes are needed for good adjoint computation.
- Slides 8-10: AMR clusters near sonic curves (especially $P1$).
  - Stagnation pressure adaptation does *not* actually care about or resolve the bow shock past the sonic line.
  - Stagnation enthalpy-based adaptation *does* resolve the bow shock all the way to the outflow..
- Slide 13: Troubleshooting convergence failure on Grid 7! Direct solve for linear, so no preconditioning. Marshall and Ben suspect that the *nonlinear* solve is preventing convergence on some grids, specifically suggesting that the shock detectors could be toggling too many times. Not sure.

4.1.1 Slide 15: Hershey-kiss stagnation point error

Total pressure p_t contours bifurcate around recirculating streamlines on unstructured meshes. This is called the "Hershey-kiss stagnation point" and it's completely unphysical.

- Scott Murman says "There's too much artificial viscosity." (he always says that)
- Ben Couchman says "too little artificial viscosity."
- Steve(n Allmaras?) says "there are multiple solutions." i.e. phase-space bifurcation.

Even without the recirculation, the stagnation point tends to float.

4.2 Jean-Marie Le Gouez

ONERA code named NXO.

- Non-compact FV. HO via weighted least-square reconstruction over a wide stencil.
- Linear combo of discrete field. 1D directional for the quads used in his simulation of CI1.
- Artificial dissipation has its own order. Most often second. They've tried fourth-order.
- $k = 2$, 3 and 5 stencils, corresponding to the order of their reconstruction. Has broad equivalence to comparable polynomial order (i.e., $k = 3 \to P = 3$).

Best-in-show results by almost a full order of magnitude.

4.3 Andrew Corrigan

DG-FEM extension of CG-FEM (originally a code for predicting noise in jets). MDG-ICE (Moving DG with Interface Condition Enforcement).

- Conservation law and interface conditions enforced separately. Space-time DG.
- Shock-fitted via standard edge collapse operations on simplices; not shock capturing.
- Gets jumps in derivatives at heads and tails of expansion fans.
- very inefficient implicit solvers at the moment.

They can't apply it to the Gaussian bump yet. They do compare to a residual-based shock capturing scheme added to their plain old DG code (not MDG-ICE). MDG-ICE gets far lower error than DG+shock capturing, and even faster-than-second-order convergence. Stagnation point pressure gets worse then better as DOF increase. Only converged the explicit DG+cap solver to four significant figures.

AMR for MDG-ICE gets wavy between the shock and the surface. MDG-ICE looks good even at $P = 2$ on isoparametrically deformed quads, but might be suffering hershey-kiss (such contours arise in error under global refinement).

4.4 Matthew Zahr

At LBNL working with Per-Olof Persson. Shock-fitted DG means Riemann solver meets all your needs. PDE-constrained optimization to solve for a shock-fitted mesh. Minimize an objective function $f(u, x)$ subject to constraint $r(u, x)$. Their objective function is supposed to be minimum when mesh faces are located at and aligned with shocks.

- Slide 4: Residual-based and Rankine-Hugoniot objective functions both have false positives away from actual shock conditions.
- Slide 5: Need fully simultaneous approach because you can't do a separate DG solve of the governing conservation law on *any* mesh that is not shock-fitted without artificial viscosity (and they don't add any artificial viscosity). Right now they use the very general (and expensive) SNOPT framework for optimization.
- Slide 6: Truly high-order convergence to shocks in Burgers equation.
- Slide 7: $P = 2$ give good picture norm with only 102 $Q = 2$ elements. It plows through a few mesh tangles. Unlike shock-capturing DG, you need an **excellent** Riemann solver. Jumps are NOT penalized like in shock-capturing DG.

4.5 Summary

Galbraith. We need to maintain distribution function across meshes.

5 CL1: Heaving and pitching airfoil

Almost perfect. No more impulsive starts or stops. Formulas close to the optimal versions of each of three cases: heaving, pitching and extractive pitching.

5.1 Per-Olof Persson

- Slide 7: Circular domain 100 chords in diameter (far field).
- Slide 8-19: Self-convergence (convergence assessed with respect to fine-mesh solution).
- Slide 8: $P2$ for Case 2 had error increase at small h.
- Slide 20: Three groups self-converge to three answers only within a few percentage points, even with steady-state solutions converged to six significant figures.
- Slide 29: Nobody bothered doing it in OpenFOAM, so Per did it in NCSU with $P1$ DG, same 3rd-order DIRK, same ALE.

Moving adiabatic walls with conserved variables and DG still do not have a boundary condition that everybody likes. Glasby mentioned Rider's studies of the wall-heating eror here.

5.2 Fidkowski

Output functions as inner product between function o and flux functions at wall.

- Slides 7-10 Tests of isentropic vortex VI1 and bump on fixed and moving deformed meshes.
- Oscillations about 10^{-4} in drag on the bump improve under P-refinement. $P4$ has far lower oscillation in drag due to mesh metion then $P3$.
- Slide 11-12: similar conclusion for moving deformed mesh tests of viscous flat-plate boundary layer.
- Slides 14-15: Checked to see if work done on airfoil (surface integral over boundary) equaled total energy deficit in flow field (volume integral). Success.
- Slide 16: had time steps empirically chosen to keep temporal error low. CFL¿1 because of ESDIRK.

Flow aligning case 2 suffered some convergence problems at $P3$.

There was a question about blended functions and geometric conservation law (GCL). Rigid overresolved, high-order simulations aren't close to blowing up in a single cycle (but it eventually will).

For the same input parameters and orders, they've got the same answers from pretty wildly different meshes that give the same answers to four significant figures. "But our errors are in the second decimal place; classic mesh dependence is not causing our problem."

Viscous runs are usually more forgiving.

6 Steve Karman. Meshes for HiOCFD

Evaluates Jacobians on "linear sub-elements within elements" and gets weighted condition number from norms. Acute angles between element edges can define inflected curve edges that drive Jacobian negative, especially in near-wall elements. You need tight tolerance.

Slide 30: Rotor 67 has a kink that needs to be fixed. Projecting rotor meshes to shroud on the other side of gaps actually helps the mesh. This works for gaps between flap sections and fuselage as well.

He finds every singularity in the geometry definition (like wingtips and intersections). The better your mesh of linear elements, the less trouble you will have curving it/them.

They try to avoid pyramids in tetrahedral meshes, but sometimes they are unavoidable.

Gaps between flap sections can cause issues with smoothers.

Good meshes have Volume > 0, Jacobian > 0, and included angles that are not too obtuse. Tetrahedra in BL are usually have far too high of an aspect ratio. They are flat and have extreme angles.

7 CR1: DPW6 Common Research Model

RANS wing-body configuration. DPW6 required fixed lift coefficient,requiring iterating on complete solutions to converge α. Marshall fixed α to avoid such repetition to match an input parameter.

Meshes are not suffciently close to one another topologically to evaluate convergence. i.e. Mesh spacing in the "Coarse" mesh is not parameterized by a single number that is some multiple of the value of such a number in the "Fine" mesh.

7.1 Galbraith

$P1Q1$ converged better than expected. $P2Q2$ however did not. 10M DOF had a count more drag ($\Delta C_D = 1e-3$) than 47M DOF.

7.2 Ryan Glasby

DPW material presented here. SUPG guys whose code is named COFFE still getting good drag with far fewer DOF.

SUPG is strong form. No DOF duplication, which is a major cost savings.

SAQCR prevents a separation bubble from interacting with a transonic shock and growing.

7.3 Behzad Ahrabi

I have no notes from this talk.

7.4 Marshall Galbraith

DG just not worth it for RANS like this. Well, DG at $P1$ certainly isn't. John said it's unusual for coarse grids to have lower drag than finer meshes. John said if you are in the linear range of C_L-α fixing C_L may not be as hard as you think.