Explicit Linking of Mathematics in the Undergraduate Engineering Curriculum

Douglas Allaire and Karen Willcox
Department of Aeronautics & Astronautics, MIT

Summary

In the fall semester 2004, a pilot program to create explicit links between engineering courses and upstream mathematics courses was implemented in Principles of Automatic Control (course 16.06). A lecture-by-lecture mapping was created that detailed the specific mathematical skills required in each 16.06 lecture and the associated upstream mathematics course where the concept was previously introduced or taught. A set of supplementary mathematics notes was developed to provide students with remedial resources for self-study and reference. Lecture content was also modified to incorporate "flashbacks", or specific references to materials used in upstream mathematics courses.

Background

In undergraduate engineering classes, the expectation is to utilize mathematical concepts that are taught in freshman and sophomore year math subjects. In the Department of Aeronautics and Astronautics, the level of mathematics skills of sophomores and juniors has been identified as a problem by a number of the faculty members that teach core subjects in the department. This issue has been quantitatively observed in the past few years, both through student performance in quizzes and final exams, and by the use of "Muddy Point cards"[1]. Often, Muddy cards contain questions about basic mathematical operations that were performed throughout the lecture. This is concerning, since if the students are stumbling on the mechanics of the problem, it is unlikely that they are grasping the true underlying physical principles and central material of the course.

The work described here is based on an initial study that was performed during 2002 and 2003 [1]. The key objective of that research was to identify barriers to deep mathematical understanding among engineering undergraduates. Data from engineering course syllabi and interviews with engineering and mathematics faculty were combined to form an implicit mathematics curriculum, which lists the mathematical skills relevant to core Aeronautics & Astronautics classes, along with the flow of learning and utilization. Interview results showed that many engineering faculty members have an inadequate knowledge of mathematics class syllabi, and often do not know where or how the skills they require are taught, while mathematics instructors often have a limited understanding of how mathematical concepts are applied in downstream engineering classes.

The major findings of the study are summarized in [1]. Table 1 shows the arrangement of topics in the implicit mathematics curriculum, along with a summary of the engineering classes where the particular skills are taught, reviewed and/or utilized. For each topic, detailed data have been compiled that describe precisely the *specific skills* needed for each mathematical concept as cited by the engineering faculty, *examples and applications* from engineering class lectures, homework and exams, *background* assumed by the engineering teaching faculty, *issues* arising either from comments by faculty or by gaps of instruction found during the analysis, and *resources and recommendations*.

---

[1] Muddy Point cards have been used extensively in Course 16 core courses. The basic idea is that small cards are distributed at the end of the lecture, and the students (anonymously) write down the muddiest point for them. The lecturer then collects these cards, and attempts to address the issues, either explicitly in the following lecture, or on a website.

Table 1: The Implicit Mathematics Curriculum. The key for courses is as follows: Fl=Fluids, Dy=Dynamics, Th=Thermodynamics, SS=Signals and Systems, Co=Controls, S=sophomore class, J=junior class (from [1]).

Mathematical Knowledge	*Utilized*	*Reviewed*	*Taught*
1 Calculus			
1.1 Functions	Fl-S, Th-S, Dy-S, SS-S, Th-J, Co-J		
1.2 Differentiation	Fl-S, Th-S, Dy-S, SS-S, Th-J, Co-J		
1.3 Integration	Fl-S, Dy-S, SS-S, Th-J, Co-J	Th-S	
1.4 Series and sums: Taylor, Fourier	Fl-S, Dy-S, SS-S, Th-J, Co-J		
1.5 Vector Calculus	Fl-S	Fl-S	
2 Geometry			
2.1 Analytical Geometry	Dy-S		
2.2 Trigonometry	Dy-S, Fl-S		
3 Differential Equations			
3.1 ODEs	Th-S, Th-J	Dy-S, Co-J	SS-S, Co-J
3.2 PDEs	Th-J	Th-J	Fl-S
3.3 Integral Equations			Fl-S
4 Linear Algebra			
4.1 Matrix Algebra	SS-S, Co-J	Dy-S, SS-S	
4.2 Linearization, Linear Systems		Co-J	Dy-S
4.3 State (discrete)			SS-S, Co-J
4.4 Tensors (multidimensional objects)			Fl-S
5 Complex Analysis			
5.1 Complex Variables	Co-J	Fl-S	SS-S
5.2 Frequency domain, variables and plots			SS-S, Co-J
5.3 Transforms: Fourier, Laplace	SS-S	Co-J	SS-S, Co-J
6 Probability and Statistics	*To be completed*		
7 Discrete Mathematics	*To be completed*		

Several problematic areas were identified, including the concept of a function, linearization, and vector calculus, and a number of recommendations were made. Lack of communication and limited pedagogical linkages between mathematics and engineering departments were found to be major contributing factors to many of the problems. The work undertaken in the fall semester of 2004 in 16.06 was a first attempt to address some of these issues. In particular, the findings from [1] were used to establish explicit linkages between mathematics and engineering courses, on a class-by-class basis.

Approach

As a first step, we created an explicit mapping between the mathematical concepts in 16.06 and upstream mathematics classes. For each lecture in 16.06, we compiled a list of the specific mathematical topics that are used, reviewed and/or taught in that lecture. The topics were organized according to the classifications used in [1] and shown in Table 1. For each topic in each lecture, we identified the *specific skills* needed for each mathematical concept, *examples and applications* found in that lecture, and an appropriate set of *resources*. Note that these data are structured in the same way as the higher-level data collected in [1], but at a much greater level of detail. For each mathematical concept, we found the specific upstream mathematics course where the concept was previously introduced or taught. The resource list was compiled to be consistent with the upstream courses, and included references to textbooks and lecture notes in the mathematics classes.

Next, we created a document, organized in a lecture-by-lecture format that reintroduces mathematical tools as they are used in 16.06. Each section of the document contains a list of skills that are expected for that lecture, some notes for selected topics, and a list of references. When citing references to other classes, we tried to be as specific as possible, e.g. referring to a particular lecture number in 18.01, 18.02, and 18.03. Each set of notes was distributed before the corresponding lecture. Students were asked to review the math notes before class, and if they felt it was necessary, review the suggested resources. Appendix A shows a sample set of these supplementary mathematics notes.

Lecture content was also modified to take advantage of the lecture-by-lecture mapping. This was done through the use of "flashbacks" – an example, an applet, or a reference to a specific topic in an upstream course. For example in Lecture 2 we consider a first-order ordinary differential equation for the roll dynamics of an aircraft. During the lecture, we referred specifically to the solution method taught in 18.03, using online 18.03 resources as guidance.

Due to the amount of time that it took to create the lecture-by-lecture mapping, we did not implement as many "real-time flashbacks" in lecture as we would have liked. This is an area of further development that will be pursued in the future as described in more detail below.

Assessment

Students were given a self-assessment quiz at the beginning and the end of semester. They were asked to rate themselves on the 16 topic areas shown in Table 2, using the following rating scale:
1 = Poor understanding, or never heard of concept
2 = Weak understanding, probably couldn't apply it properly
3 = OK understanding, could apply it with considerable effort
4 = Good understanding, could apply it with little or no trouble
5 = Excellent understanding, almost second nature

Table 2: Topics for self-assessment quiz.

Topic Number	Topic
1	The concept of a function
2	Linearize a nonlinear system about an operating point
3	Perform a Taylor series expansion of a function about a point
4	Understand the implications of linearizing a nonlinear system
5	Convolution integral
6	Perform an inverse Laplace transform
7	Reduce a second-order ODE to a system of first-order ODEs
8	Solve a first-order, linear, constant coefficient ODE
9	Partial differentiation of multivariate functions
10	Final value theorem
11	Convert a complex number from Cartesian to polar form
12	Plot a complex number
13	Complex conjugation
14	Perform a partial fraction expansion using any technique
15	Perform a partial fraction expansion using the cover-up method
16	Compute the eigenvalues and eigenvectors of a second-order system

The results of the self-assessment quizzes are shown in Figure 1. It can be seen that after completing 16.06, the students rated themselves more highly on every topic. The post-16.06 ratings all have average values of 3 or greater; 12 of the 16 topics averaged scores higher than 4. It is not surprising that students' level of comfort would increase as a result of taking 16.06, as the mathematic topics in Table 2 are utilized and/or reviewed in detail in the course. Unfortunately self-assessment data from previous years is not available, so it is difficult to discern whether the new implementation of the mathematics resources and flashbacks was a contributing factor to the positive increase. It is also interesting to note that the topic on which the students rated themselves most highly, the concept of a function, was one of the major areas of weaknesses identified by the engineering course instructors in the study in [1].

As part of the course survey given at the end of the semester, the students were asked two questions regarding the supplementary math notes that were distributed. The response rate of the survey was 33 out of 62 students. Of the 33 responses, 64% of students responded that they did not use the notes; 36% of students responded that they did use the notes. When asked to comment on the effectiveness of the notes, 18% rated the notes as *not effective*, 42% rated them as *somewhat effective*, 9% rated them as *very effective* and 30% said that they did not use them.. (Note the inconsistency between the responses to the two questions.) In the essay part of the survey, the responses to the notes ranged from "Really unnecessary" to "I couldn't have survived the first part of the course without them". In the detailed survey comments, the students made some good suggestions for improvements in future years. Several students remarked that the reference math resources were difficult to obtain. One student commented "the supplementary math notes are very minimalist. It would be more helpful if they were fleshed out to more than just an outline". Future actions to address these comments are described below.

Throughout the semester, it appeared to the instructors (Deyst and Willcox) that the mathematics notes and the linkages made in lecture were serving their intended purpose. In particular, we noted many fewer questions relating to basic mathematics concepts and skills during the lectures and on muddy cards. We included targeted math questions on each homework assignment – student performance on these questions was excellent.

Recommendations and Future Steps

Even though the student evaluations indicated that the students did not rate the mathematics notes as being overly effective, we believe that class performance shows that the combination of these notes and an increased emphasis on linkages during lecture helped considerably with students' grasp of underlying mathematical concepts. The supplementary notes appear to be an important resource – not for all students, but for those students who struggle with the underlying mathematics. These strategies will be continued in future years and we will continue to build upon and expand the supplementary notes.

Most of the TA resources in the fall of 2004 were spent performing the lecture-by-lecture mapping – this turned out to be a more time-intensive task than anticipated. As a result, we did not implement as many "real-time flashbacks" in lecture as envisioned. Further development of this aspect is planned for future years. In particular, we plan to work with MIT IS&T to explore better use of educational technology in this area. For example, Prof. Haynes Miller of the Department of Mathematics is creating a set of indexed video lectures for 18.03. One idea for the future would be to play a snippet of 18.03 video during a 16.06 lecture to remind students explicitly of the mathematics they learned in previous years.

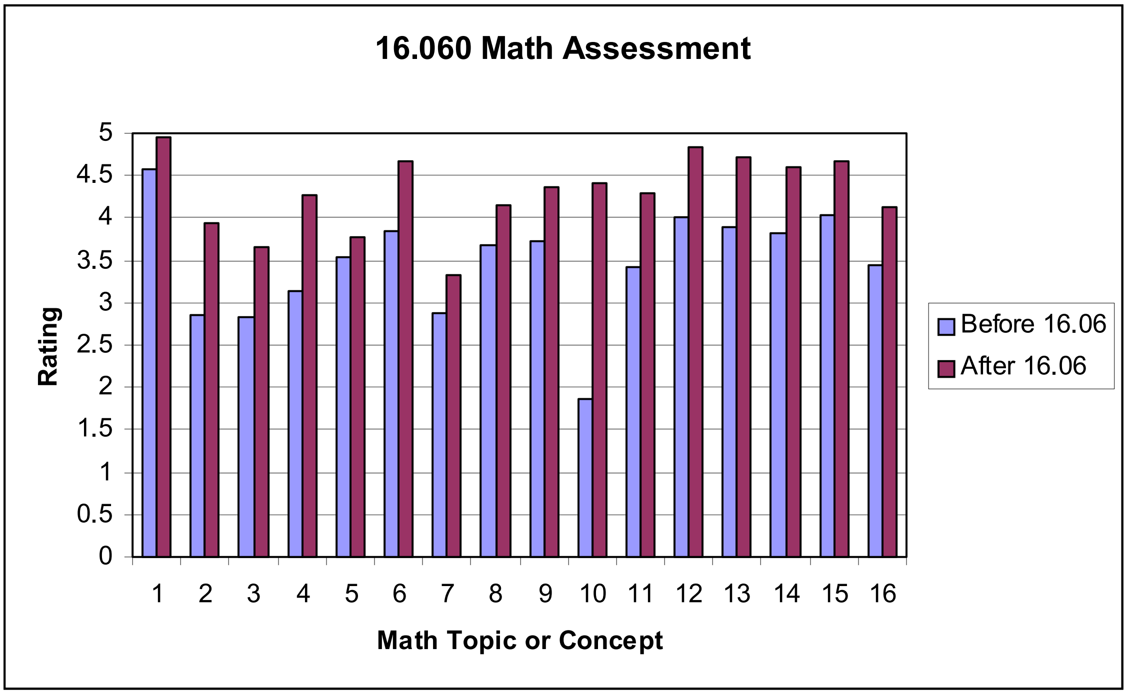

Figure 1: Average ratings for self-assessment quizzes given at the beginning and the end of the semester. The topic numbers refer to the topics given in Table 2.

Several students commented that the mathematics resources were difficult to access. This problem has been addressed to some extent since 18.01 and 18.03 have both been published on MIT OCW. To make the resources even more accessible, we are working with OCW staff to create a living document from the supplementary mathematics notes, i.e. where possible, the references in the notes appear as links that take the students directly to the relevant mathematics website. The vision is also to create forward links – where mathematics course notes could link to engineering course websites in order to show students the relevant engineering applications of the mathematics they are learning.

Acknowledgements

This work was supported by the Class of 1951 Fund for Excellence in Education, the Class of 1955 Fund for Excellence in Teaching, the Class of 1972 Fund for Educational Innovation, and the Class of 1999 Fund for Excellence in Student Learning.

References

[1] Willcox, K. and Bounova, G., "Mathematics in Engineering: Identifying, Enhancing, and Linking the Implicit Mathematics Curriculum," in Proceedings of American Society for Engineering Education Annual Conference & Exposition, Salt Lake City, UT, June 2004.