Engineering Sketch Pad (ESP) Training
Session 6: Sensitivities

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Revised for v1.11

Overview

- Background / Objective
- Alternative approaches
  - analytic derivatives
  - code differentiation
  - finite differences
- Computed examples
- Application to grid generation
- Conclusions
- Computing sensitivities in ESP
- Hands-on exercises

Background/ Objective

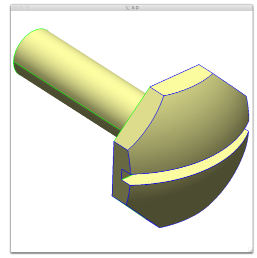
- Background
  - MDAO environments require calculation of sensitivity of objective function(s) w.r.t. the design parameters
  - Many modern CFD systems can produce the objective function sensitivity all the way back to the grid
  - Little work has been done in calculating the sensitivity of the grid w.r.t. the design parameters
- Objective
  - Compute sensitivities directly on parametric, CAD-based geometries

Possible Approaches

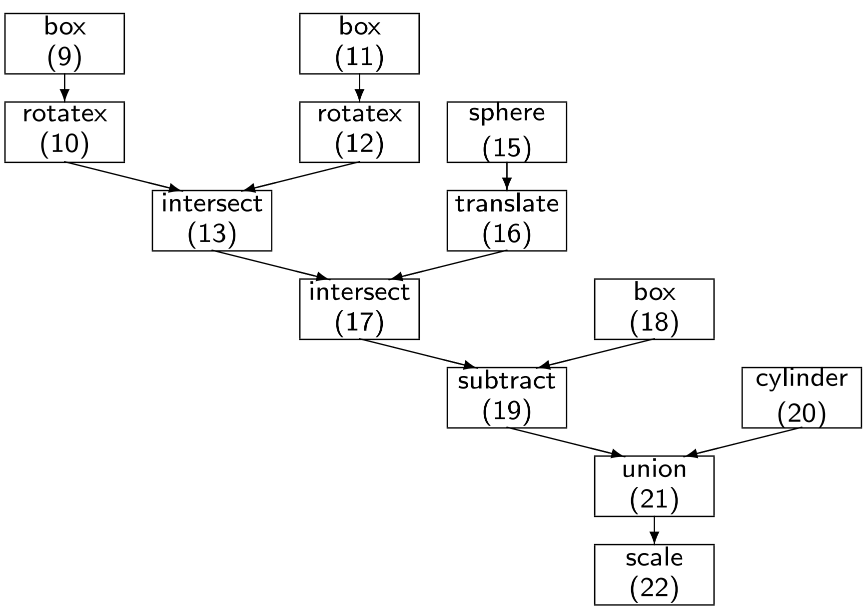
- Analytic derivatives
  - differentiate all operations within the CAD system analytically
  - requires access to CAD system's algorithms
  - produces results that are not susceptible to truncation error
- Code differentiation
  - CAD system source code is automatically differentiated via compiler-like process
  - requires access to CAD system's source code
  - produces results that are not susceptible to truncation error
- Finite differences
  - multiple instances of the configuration are generated and the sensitivities are computed via finite differences
  - requires one to find corresponding points in the configurations
  - picking appropriate step size (or perturbation) requires a trade-off between truncation and round-off errors

Review of Construction Process (1)

```
      # bolt example

      # design parameters
 1:   despmtr   Thead     1.00   # thickness of head
 2:   despmtr   Whead     3.00   # width     of head
 3:   despmtr   Fhead     0.50   # fraction  of head that is flat

 4:   despmtr   Dslot     0.75   # depth of slot
 5:   despmtr   Wslot     0.25   # width of slot

 6:   despmtr   Lshaft    4.00   # length   of shaft
 7:   despmtr   Dshaft    1.00   # diameter of shaft

 8:   despmtr   sfact     0.50   # overall scale factor

      # head
 9:   box       0      -Whead/2  -Whead/2  Thead   Whead     Whead
10:   rotatex   90  0  0
11:   box       0      -Whead/2  -Whead/2  Thead   Whead     Whead
12:   rotatex   45  0  0
13:   intersect

14:   set       Rhead  (Whead^2/4+(1-Fhead)^2*Thead^2)/(2*Thead*(1-Fhead))

15:   sphere    0              0   0    Rhead
16:   translate Thead-Rhead    0   0
17:   intersect
```



Review of Construction Process (2)



Analytical Sensitivities

- Differentiate expressions for arguments to various operators
- For each Face
  - determine primitive that created the Face
  - differentiate functions used to generate the Face in its original position
  - apply appropriate transformations to sensitivities
- For each Edge
  - compute sensitivities of adjacent Faces
  - find sensitivity that is consistent with them and whose component along the Edge vanishes
- For each Node
  - compute sensitivities of incident Edges
  - find sensitivity that is consistent with them

Analytical Sensitivity for Faces

- Differentiate function(s) used to create a point on the Face
  - for a box

$$\left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{prim}} = \frac{\partial \vec{x_0}}{\partial P} + \frac{\partial \vec{S}}{\partial P}\left(\frac{\vec{x}_{\text{prim}} - \vec{x_0}}{\vec{S}}\right)$$

- Modify the sensitivities based upon transformations traversed in the feature tree
  - for a translation

$$\left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{new}} = \left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{prim}} + \frac{d\vec{x_0}}{dP}$$

- Take normal component

$$\frac{\partial w}{\partial P} \equiv \frac{\partial \vec{x}}{\partial P} \bullet \vec{n}$$

Analytical Sensitivity for Edges and Nodes

- Edge sensitivity is consistent with the adjacent Face sensitivities

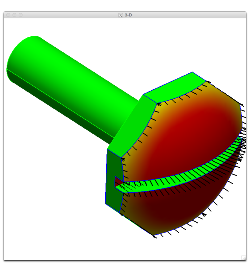
$$\begin{bmatrix} n_{x,\text{left}} & n_{y,\text{left}} & n_{z,\text{left}} \\ n_{x,\text{right}} & n_{y,\text{right}} & n_{z,\text{right}} \\ t_{x,\text{edge}} & t_{y,\text{edge}} & t_{z,\text{edge}} \end{bmatrix} \begin{bmatrix} (\partial x/\partial P)_{\text{edge}} \\ (\partial y/\partial P)_{\text{edge}} \\ (\partial z/\partial P)_{\text{edge}} \end{bmatrix} = \begin{bmatrix} (\partial w/\partial P)_{\text{left}} \\ (\partial w/\partial P)_{\text{right}} \\ 0 \end{bmatrix}$$

- Node sensitivity is consistent with the incident Edge sensitivities

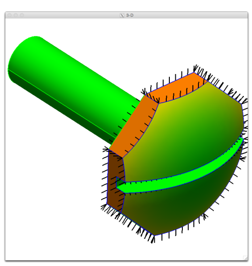
$$\begin{bmatrix} \vec{t}_1 \bullet \vec{t}_1 & -\vec{t}_1 \bullet \vec{t}_2 \\ -\vec{t}_1 \bullet \vec{t}_2 & \vec{t}_2 \bullet \vec{t}_2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} ((\partial \vec{x}/\partial P)_2 - (\partial \vec{x}/\partial P)_1) \bullet \vec{t}_1 \\ ((\partial \vec{x}/\partial P)_1 - (\partial \vec{x}/\partial P)_2) \bullet \vec{t}_2 \end{bmatrix}$$

$$\left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{node}} = \left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{edge1}} + A\left(\frac{\partial \vec{x}}{\partial t}\right)_{\text{edge1}}$$

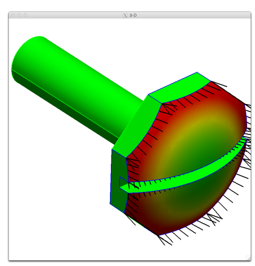
Analytical Sensitivities Example



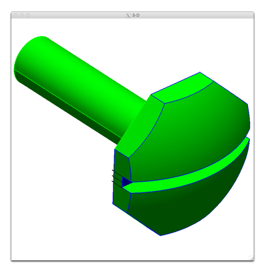
$\partial \vec{x}/\partial$**(Thead)**



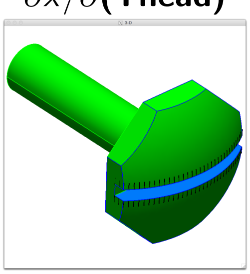
$\partial \vec{x}/\partial$**(Whead)**



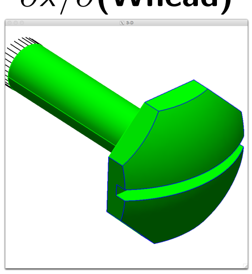
$\partial \vec{x}/\partial$**(Fhead)**



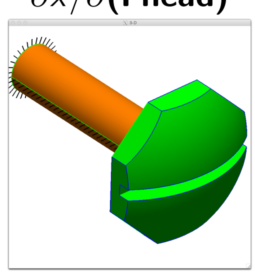
$\partial \vec{x}/\partial$**(Dslot)**



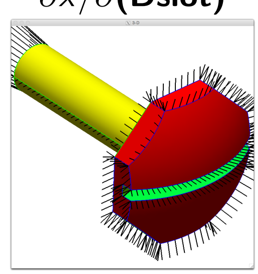
$\partial \vec{x}/\partial$**(Wslot)**



$\partial \vec{x}/\partial$**(Lshaft)**



$\partial \vec{x}/\partial$**(Dshaft)**



$\partial \vec{x}/\partial$**(sfact)**

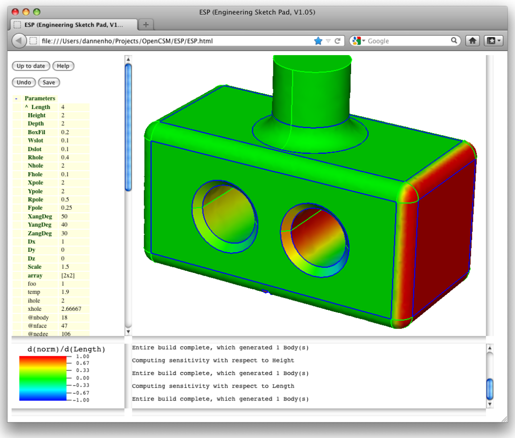
Finite-difference Sensitivities (1)

- Basic strategy:
  - re-create configuration after perturbing a design parameter
    - requires regeneration
    - step-size must be chosen carefully
  - take finite difference of associated points in the configurations
- Assumptions made in previous approaches:
  - dilitation or contraction is related to Face's bounding parametric coordinates
    - local changes have large effect on whole Face
  - geometry's parametrization can be used to map point movement
    - for NURBs, geometry is based upon knot spacings

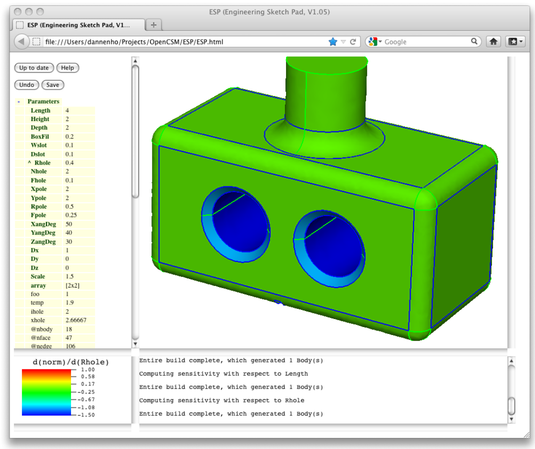
Finite-difference Sensitivities (2)

- New approach:
  - compute a tessellation in the base configuration
    - discretize the Edges first
    - fill region with triangles only using the Edge points
  - discretize the perturbed Edges
    - use relative arc-lengths
    - find parametric coordinates $\vec{u}$ for adjacent Edges using "Pcurve" evaluations ($\vec{u}(t)$)
    - compute perturbation of space coordinates $\vec{x}$ on the Edges
  - for interior points
    - find barycentric coordinates in base coarse tessellation
    - propagate Edge parametric coordinate perturbations from the Edges to the interior
    - compute perturbation of space coordinates

Finite-difference Sensitivity Example (1)

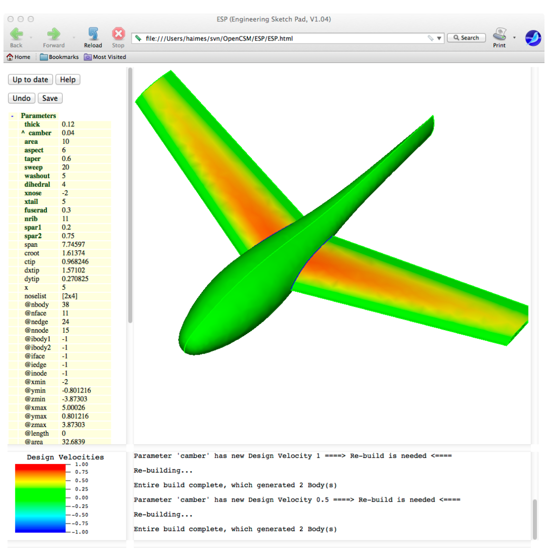


Change in box length

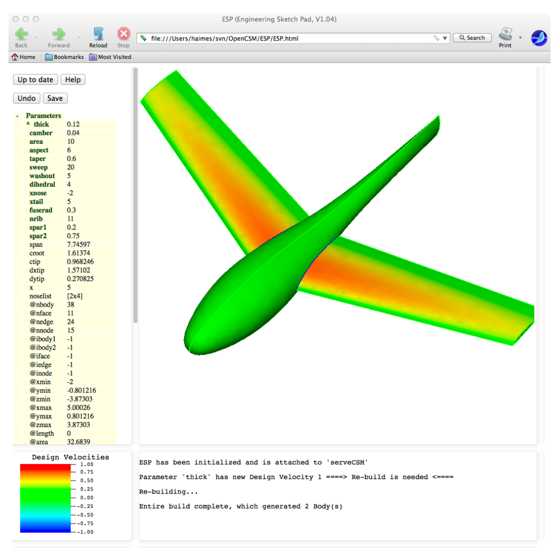


Change in the holes' radii

Finite-difference Sensitivity Example (2)



Change in camber



Change in thickness

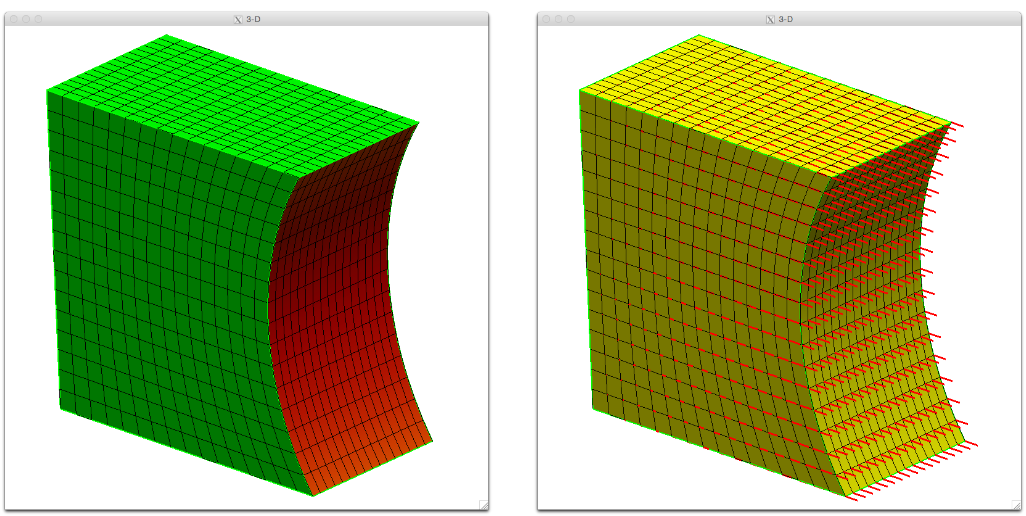
Application to Grid Sensitivities

- Use configuration sensitivities to find (normal) change to surface location
- Use derivative of grid generator to find tangential change along surface

$$\left(\frac{d\vec{x}}{dP}\right)_{i,j} = \left(\frac{\partial w}{\partial P}\right)_{i,j} \vec{n}_{i,j} + \left(\frac{\partial \vec{x}}{\partial \vec{u}}\right)_{i,j} \left(\frac{d\vec{u}}{dP}\right)_{i,j}$$

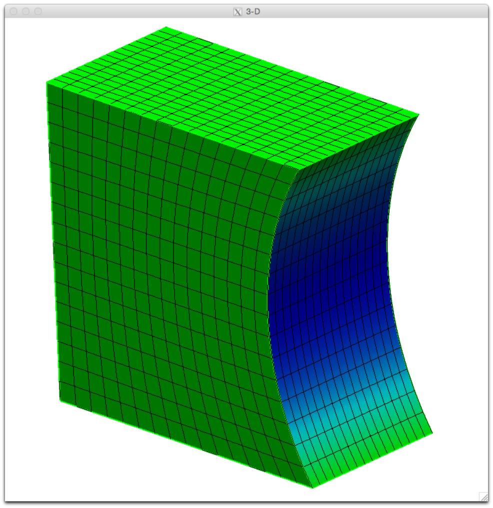
- $d\vec{u}/dP$ in the interior comes from $d\vec{u}/dP$ on the Edges, which come from $d\vec{u}/dP$ at the Nodes
- Process is easily executed by doing Nodes first, then Edges, then Faces

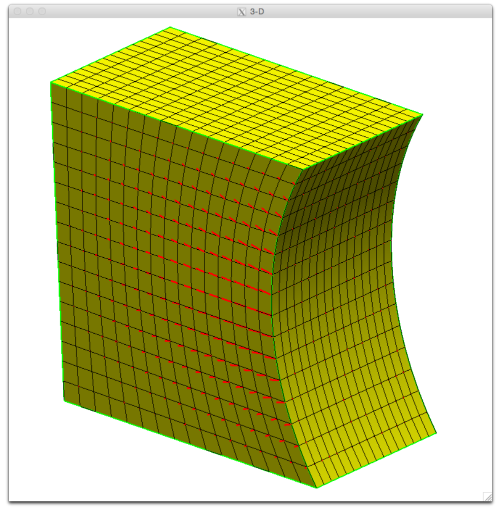
Grid Sensitivities Example (1)



Sensitivity with respect to the length of the box

Grid Sensitivities Example (2)





Sensitivity with respect to the depression distance

Conclusions (1)

- Sensitivities of a parametric, CAD-generated configuration w.r.t. design parameters can be robustly and efficiently found using a combination of techniques
  - Analytic derivatives are used whenever possible
    - efficient — do not require regeneration of configuration
    - accurate — not susceptible to truncation error
    - automatic code differentiation can be used when source code is available and derivatives are too hard to compute by hand
  - Finite differences are used when necessary
    - require regeneration of perturbed configuration
    - the original tessellation is reused to ensure proper point matching between base and perturbed geometries

Conclusions (2)

- Configuration sensitivity is computed locally based upon a point's initial location in space
  - returns motion normal to Faces and Edges
  - is insensitive to surface parametrization
- Grid sensitivities can be found using just the configuration sensitivities and a knowledge of the grid generation scheme
- Tools are now available to produce the sensitivity of MDAO objective function(s) w.r.t. the engineer's design parameters

Computing Sensitivities in ESP (1)

- Build a model with Design Parameters
- For simple sensitivities (that is, with respect to one Design Parameter at a time)
  - select (edit) the Design Parameter
  - press **Compute Sensitivity**
  - configuration will automatically be rebuilt and display will change
    - minimum and maximum sensitivities will be reported in Messages window
    - configuration will be colored in Graphics window
    - Key window will contain the color key, whose limits can be changed by clicking in the Key window

Computing Sensitivities in ESP (2)

- The meaning of the various colors is:
  - red (positive sensitivity) are regions where a positive change in the Design Parameter would move the surface in the direction of the local outward-facing surface normal
  - blue (negative sensitivity) are regions where a negative change in the Design Parameter would move the surface in a direction opposite the local outward-facing surface normal
- Example for a cylindrical feature:
  - for a post-like feature, the sensitivity with respect to the diameter would be positive
  - for a hole-like feature, the sensitivity with respect to the diameter would be negative

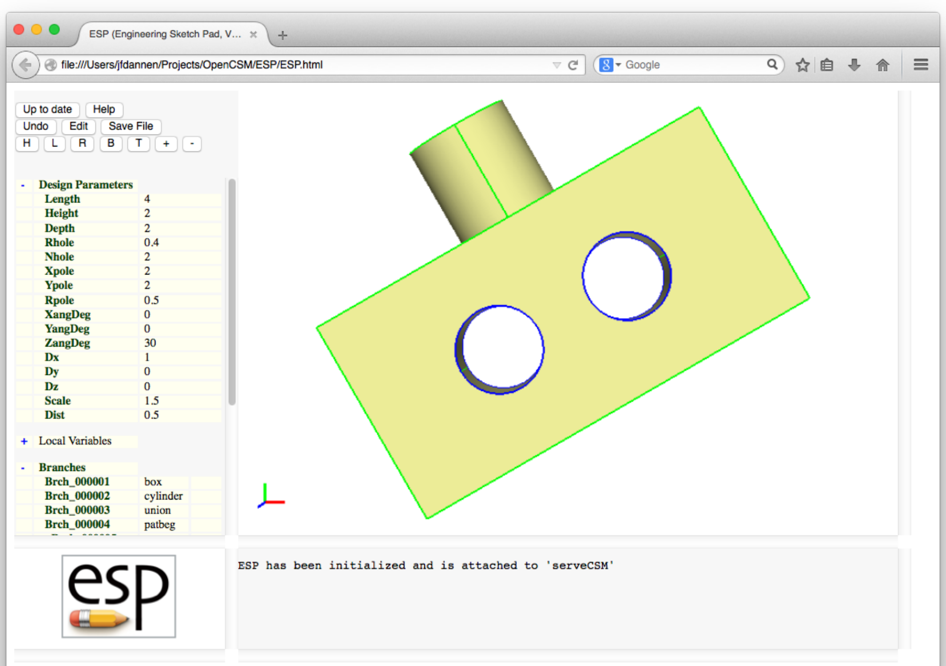
Computing Sensitivities in ESP (3)

- To find the sensitivity with respect to a multi-valued Design Parameter
  - select (edit) the multi-valued Design Parameter
  - press **Clear Design Velocities**
  - press **Set Design Velocity**
    - answer **1** for the entity for which you want the sensitivity
    - answer **0** (the default) for all other entities

Computing Sensitivities in ESP (4)

- To find the sensitivity with respect to a several Design Parameters at the same time (for example, in the direction of the gradient proposed by an optimizer)
  - select any Design Parameter
  - press **Clear Design Velocities**
  - for each Design Parameter whose component to the gradient direction is non-zero
    - press **Set Design Velocity**
    - enter the associated component of the gradient vector
  - press **Press to Re-build**
  - Note: the key window will say `d(norm)/d(***)` to indicate that the sensitivity is with respect to some combination of Design Parameters

Simple Block (1)



Simple Block (2)

Box		
Length Height Depth	length of box height of box depth of box anchored at $X = Z = 0$ centered at $Y = 0$	4.0 2.0 2.0
Holes		
Rhole Nhole	radii of the holes number of holes holes are equally spaced	0.4 2
Pole		
Xpole Ypole Rpole	X-location of top of pole Y-location of top of pole radius of pole	2.0 2.0 0.5

Simple Block (3)

Rotation about origin		
XangDeg YangDeg ZangDeg	X rotation (deg) Y rotation (deg) Z rotation (deg)	0. 0. 30.
Translation		
Dx Dy Dz		1.0 0.0 0.0
Scaling		
Scale	overall scaling factor	1.5

Simple Block (4)

- What is the sensitivity to each Design Parameter?
- What is the sensitivity if you change two Design Parameters at the same time?

Muddy Cards

- Any questions?
- Any suggestions?
- Were the examples useful?