Engineering Sketch Pad (ESP)

Training Session 9
Sensitivities

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updated for v1.19

Overview

- Background / Objective
- Alternative approaches
  - analytic derivatives
  - code differentiation
  - finite differences
- Computed examples
- Application to grid generation
- Computing sensitivities in ESP
- Homework exercise

Background/ Objective

- Background
  - MDAO environments require calculation of sensitivity of objective function(s) w.r.t. the design parameters
  - Many modern CFD systems can produce the objective function sensitivity all the way back to the grid
  - Little work has been done in calculating the sensitivity of the grid w.r.t. the design parameters
- Objective
  - Compute sensitivities directly on parametric, CAD-based geometries

Geometric Sensitivities

- Geometric sensitivities tell how a point ($\vec{x}$) on a very smooth configuration would appear to move with respect to the change of any Design Parameter (P)
- For example, consider a cylinder
  - points on the curved Faces would appear to move if the cylinder's location or radius changed
  - points on the curved Faces would NOT appear to change if the cylinder's length changed
- The Geometric sensitivity just has a component normal to the Face (or Edge)

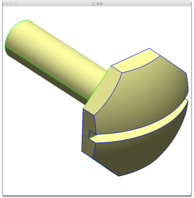
Tessellation Sensitivities

- Tessellation sensitivities tell how points in a grid or tessellation ($\vec{x}_i$) *might* move with respect to the change of any Design Parameter (P)
- For example, consider a cylinder
  - points on the curved Faces would appear to move if the cylinder's location, radius, or length changed
- The Tessellation sensitivity has components normal to and along the Face (or Edge)

Possible Approaches

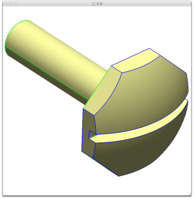
- Analytic derivatives
  - differentiate all operations within the CAD system analytically
  - requires access to CAD system's algorithms
  - produces results that are not susceptible to truncation error
- Code differentiation
  - CAD system source code is automatically differentiated via compiler-like process
  - requires access to CAD system's source code
  - produces results that are not susceptible to truncation error
- Finite differences
  - multiple instances of the configuration are generated and the sensitivities are computed via finite differences
  - requires one to find corresponding points in the configurations
  - picking appropriate step size (or perturbation) requires a trade-off between truncation and round-off errors

```
     # bolt example

     # design parameters
 1:  DESPMTR   Thead     1.00   # thickness of head
 2:  DESPMTR   Whead     3.00   # width     of head
 3:  DESPMTR   Fhead     0.50   # fraction  of head that is flat

 4:  DESPMTR   Dslot     0.75   # depth of slot
 5:  DESPMTR   Wslot     0.25   # width of slot

 6:  DESPMTR   Lshaft    4.00   # length   of shaft
 7:  DESPMTR   Dshaft    1.00   # diameter of shaft

 8:  DESPMTR   sfact     0.50   # overall scale factor

     # head
 9:  BOX       0      -Whead/2  -Whead/2  Thead   Whead     Whead
10:  ROTATEX   90  0  0
11:  BOX       0      -Whead/2  -Whead/2  Thead   Whead     Whead
12:  ROTATEX   45  0  0
13:  INTERSECT

...
```

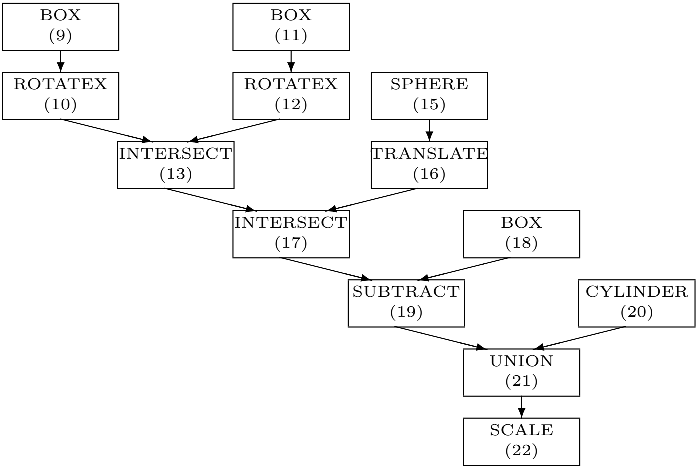


Review of Construction Process (2)

```
...

SET       Rhead  (Whead^2/4+(1-Fhead)^2*Thead^2)/(2*Thead*(1-Fhead))

SPHERE    0            0   0    Rhead
TRANSLATE Thead-Rhead  0   0
INTERSECT

          # slot
BOX       Thead-Dslot  -Wslot/2   -Whead   2*Thead   Wslot    2*Whead
SUBTRACT

          # shaft
CYLINDER  -Lshaft   0   0   0   0   0   Dshaft/2
UNION

SCALE     sfact

END
```



Review of Construction Process (3)



Analytical Sensitivities

- Differentiate expressions for arguments to various operators
- For each Face
  - determine primitive that created the Face
  - differentiate functions used to generate the Face in its original position
  - apply appropriate transformations to sensitivities
  - return the normal component
- For each Edge
  - compute sensitivities of adjacent Faces
  - find sensitivity that is consistent with them and whose component along the Edge vanishes
- For each Node
  - compute sensitivities of incident Edges
  - find sensitivity that is consistent with them

Analytical Sensitivity for Faces — Example

- Differentiate function(s) used to create a point on the Face
  - for a box (starting at $\vec{x}_0$ with a size $\vec{L}$)

$$\left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{prim}} = \frac{\partial \overrightarrow{x_0}}{\partial P} + \frac{\partial \vec{L}}{\partial P}\left(\frac{\vec{x}_{\text{prim}} - \overrightarrow{x_0}}{\vec{L}}\right)$$

- Modify the sensitivities based upon transformations traversed in the feature tree
  - for a scaling (by a factor S)

$$\left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{new}} = S\left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{prim}} + \vec{x}\frac{dS}{dP}$$

- Take normal component

$$\frac{\partial w}{\partial P} \equiv \frac{\partial \vec{x}}{\partial P} \bullet \vec{n}$$

Analytical Sensitivity for Edges and Nodes

- Edge sensitivity is consistent with the adjacent Face sensitivities (but has zero component along the Edge)

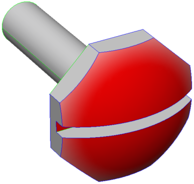
$$\begin{bmatrix} n_{x,\text{left}} & n_{y,\text{left}} & n_{z,\text{left}} \\ n_{x,\text{right}} & n_{y,\text{right}} & n_{z,\text{right}} \\ t_{x,\text{edge}} & t_{y,\text{edge}} & t_{z,\text{edge}} \end{bmatrix} \begin{bmatrix} (\partial x/\partial P)_{\text{edge}} \\ (\partial y/\partial P)_{\text{edge}} \\ (\partial z/\partial P)_{\text{edge}} \end{bmatrix} = \begin{bmatrix} (\partial w/\partial P)_{\text{left}} \\ (\partial w/\partial P)_{\text{right}} \\ 0 \end{bmatrix}$$

- Node sensitivity is consistent with the incident Edge sensitivities

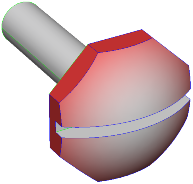
$$\begin{bmatrix} \vec{t}_1 \bullet \vec{t}_1 & -\vec{t}_1 \bullet \vec{t}_2 \\ -\vec{t}_1 \bullet \vec{t}_2 & \vec{t}_2 \bullet \vec{t}_2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} ((\partial \vec{x}/\partial P)_2 - (\partial \vec{x}/\partial P)_1) \bullet \vec{t}_1 \\ ((\partial \vec{x}/\partial P)_1 - (\partial \vec{x}/\partial P)_2) \bullet \vec{t}_2 \end{bmatrix}$$

$$\left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{node}} = \left(\frac{\partial \vec{x}}{\partial P}\right)_{\text{edge1}} + A \left(\frac{\partial \vec{x}}{\partial t}\right)_{\text{edge1}}$$

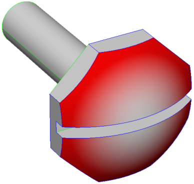
Analytical Sensitivities Example



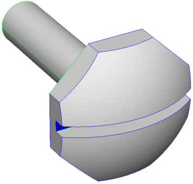
$\partial\vec{x}/\partial(\textbf{Thead})$



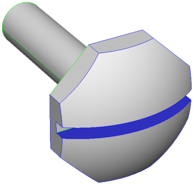
$\partial\vec{x}/\partial(\textbf{Whead})$



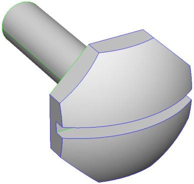
$\partial\vec{x}/\partial(\textbf{Fhead})$



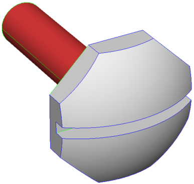
$\partial\vec{x}/\partial(\textbf{Dslot})$



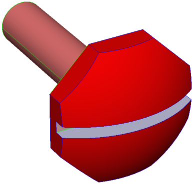
$\partial\vec{x}/\partial(\textbf{Wslot})$



$\partial\vec{x}/\partial(\textbf{Lshaft})$



$\partial\vec{x}/\partial(\textbf{Dshaft})$



$\partial\vec{x}/\partial(\textbf{sfact})$

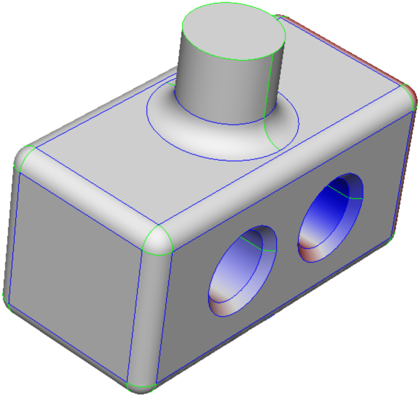
Finite-difference Sensitivities (1)

- Basic strategy:
    - re-create configuration after perturbing a design parameter
        - requires regeneration
        - step-size must be chosen carefully
    - take finite difference of associated points in the configurations
- Assumptions made in previous approaches:
    - dilitation or contraction is related to Face's bounding parametric coordinates
        - local changes have large effect on whole Face
    - geometry's parametrization can be used to map point movement
        - for NURBs, geometry is based upon knot spacings

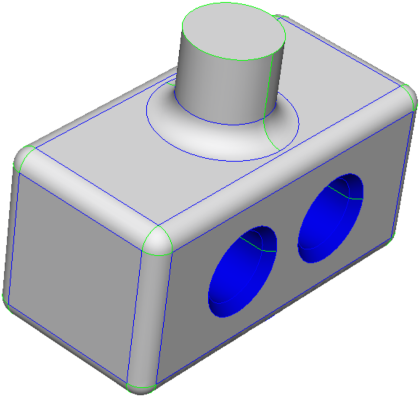
Finite-difference Sensitivities (2)

- New approach:
  - compute a tessellation in the base configuration
    - discretize the Edges first
    - fill region with triangles only using the Edge points
  - discretize the perturbed Edges
    - use relative arc-lengths
    - find parametric coordinates $\vec{u}$ for adjacent Edges using "Pcurve" evaluations ($\vec{u}(t)$)
    - compute perturbation of space coordinates $\vec{x}$ on the Edges
  - for interior points
    - find barycentric coordinates in base coarse tessellation
    - propagate Edge parametric coordinate perturbations from the Edges to the interior
    - compute perturbation of space coordinates
- See AIAA-2015-1370, available from `acdl.mit.edu/ESP`

Finite-difference Sensitivity Example (1)

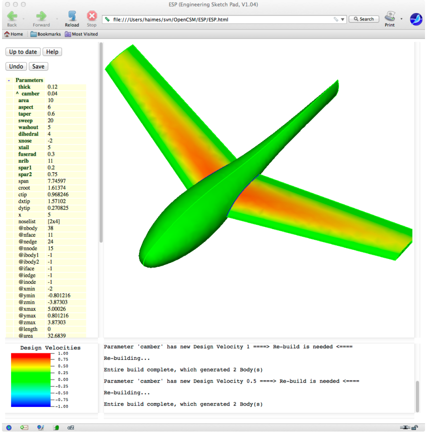


Change in box length

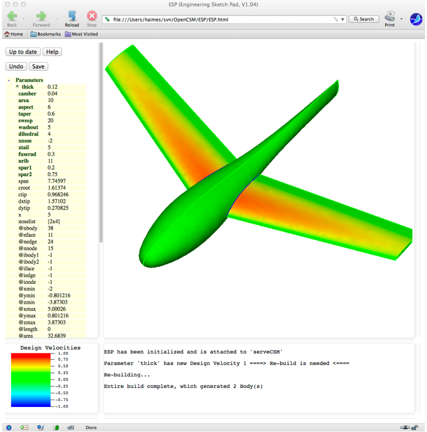


Change in the holes' radii

Finite-difference Sensitivity Example (2)



Change in camber



Change in thickness

Computing Tessellation Sensitivities

- Use geometric sensitivities to find (normal) change to surface location
- Use derivative of (surrogate) grid generator to find tangential change along surface

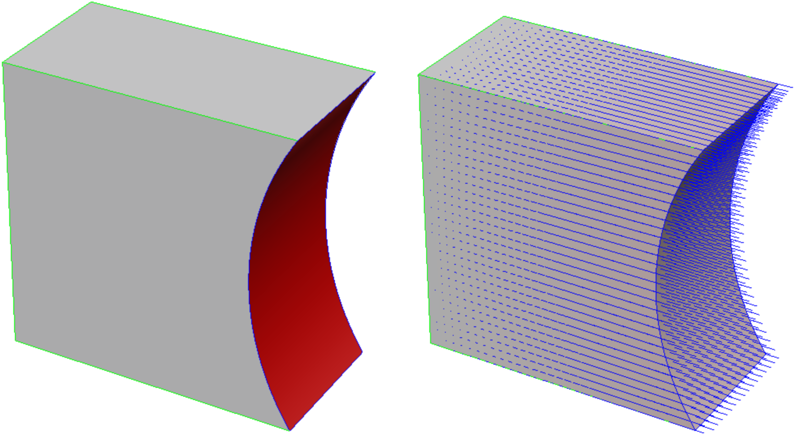
$$\left(\frac{d\vec{x}}{dP}\right)_{i,j} = \left(\frac{\partial w}{\partial P}\right)_{i,j} \vec{n}_{i,j} + \left(\frac{\partial \vec{x}}{\partial \vec{u}}\right)_{i,j} \left(\frac{d\vec{u}}{dP}\right)_{i,j}$$

- $d\vec{u}/dP$ in the interior comes from $d\vec{u}/dP$ on the Edges, which come from $d\vec{u}/dP$ at the Nodes
- Process is easily executed by doing Nodes first, then Edges, then Faces

Tessellation Sensitivities Example (1)

Sensitivity w.r.t. Length of box

Geometric sens. Tessellation sens.

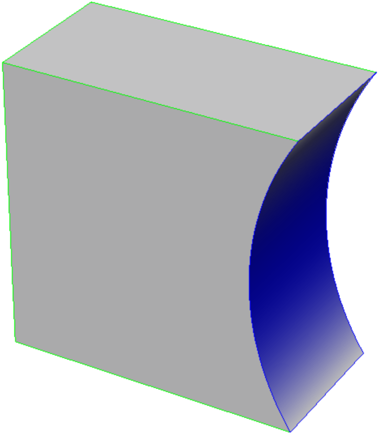


Sensitivity with respect to the length of the box

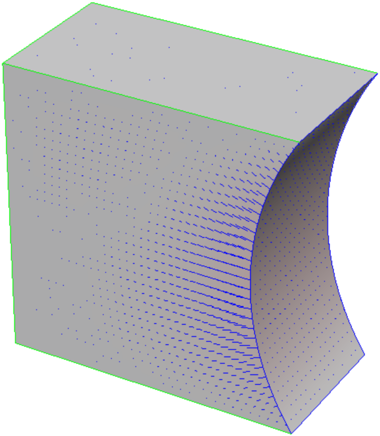
Tessellation Sensitivities Example (2)

Sensitivity w.r.t. depression radius

Geometric sens.



Tessellation sens.



Sensitivity with respect to the depression distance

Geometric Sensitivities in ESP (1)

- Build a model with Design Parameters
- For simple sensitivities (that is, with respect to one Design Parameter at a time)
  - select (edit) the Design Parameter
  - press **Compute geom sens**
  - configuration will automatically be rebuilt and display will change
    - minimum and maximum sensitivities will be reported in MessageWindow
    - configuration will be colored in GraphicsWindow
    - KeyWindow will contain the color key, whose limits can be changed by clicking in the KeyWindow

Geometric Sensitivities in ESP (2)

- The meaning of the various colors is:
  - red (positive sensitivity) are regions where a positive change in the Design Parameter would move the surface in the direction of the local outward-facing surface normal
  - blue (negative sensitivity) are regions where a negative change in the Design Parameter would move the surface in a direction opposite the local outward-facing surface normal
- Example for a cylindrical feature:
  - for a post-like feature, the sensitivity with respect to the diameter would be positive (red)
  - for a hole-like feature, the sensitivity with respect to the diameter would be negative (blue)

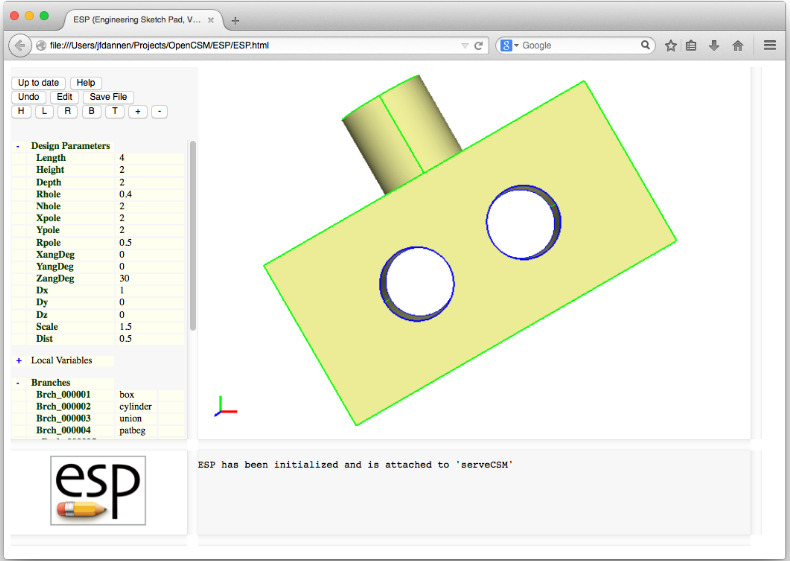
Geometric Sensitivities in ESP (3)

- To find the sensitivity with respect to a multi-valued Design Parameter
  - select (edit) the multi-valued Design Parameter
  - press **Clear Design Velocities**
  - set the velocities in the lower part of the form
    - **1** for the entity for which you want the sensitivity
    - **0** (the default) for all other entities
  - press **Compute geom sens** or **Compute tess sens**

Geometric Sensitivities in ESP (4)

- To find the sensitivity with respect to a several Design Parameters at the same time (for example, in the direction of the gradient proposed by an optimizer)
  - select any Design Parameter
  - press **Clear Design Velocities**
  - for each Design Parameter whose component to the gradient direction is non-zero, put a **1** in the velocity table(s)
  - press **Press to Re-build**
  - Note: the KeyWindow will say `d(norm)/d(***)` to indicate that the sensitivity is with respect to some combination of Design Parameters

Simple Block (1)



Simple Block (2)

Box		
Length	length of box	4.0
Height	height of box	2.0
Depth	depth of box	2.0
	anchored at $X = Z = 0$	
	centered at $Y = 0$	
Holes		
Rhole	radii of the holes	0.4
Nhole	number of holes	2
	holes are equally spaced	
Pole		
Xpole	X-location of top of pole	2.0
Ypole	Y-location of top of pole	2.0
Rpole	radius of pole	0.5

Simple Block (3)

Rotation about origin		
XangDeg	X rotation (deg)	0.
YangDeg	Y rotation (deg)	0.
ZangDeg	Z rotation (deg)	30.
Translation		
Dx		1.0
Dy		0.0
Dz		0.0
Scaling		
Scale	overall scaling factor	1.5

Simple Block (4)

- Starting file is at `$ESP_ROOT/training/ESP/data/session09/simpleBlock.csm`
- What is the geometric sensitivity to each Design Parameter?
- What is the geometric sensitivity if you change two Design Parameters at the same time?
- What is the tessellation sensitivity to each Design Parameter?