

# Computational Aircraft Prototype Syntheses



## Training Session 3

### CAPS with Gradient-based Optimization

ESP v1.26

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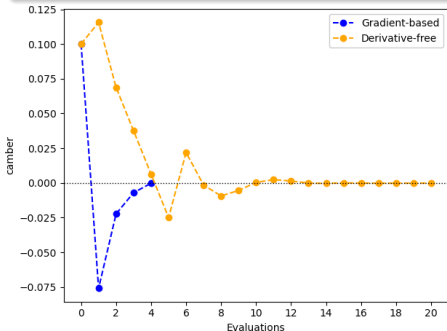
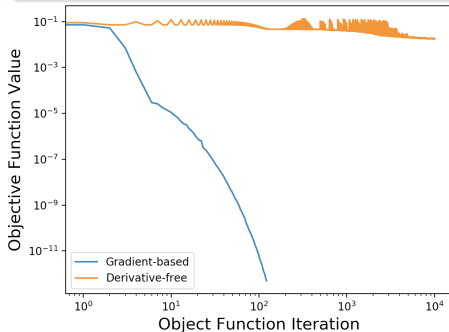
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NACA airfoil,  $\alpha = 0^\circ$  $\min_{\text{camber}} C_D$  $L^2$  minimization with 200 design parameters

- MSES Overview
- Accessing Functional Derivatives
- Gradient-Based Optimization with Analysis Input
- Gradient-Based Optimization with Geometric Inputs
- Gradient-based Optimization for Structures
- Suggested Exercises

## MSES

- Multi-Element airfoil performance analysis
- Integral Boundary Layer Theory + Euler
  - Transonic flow
- $e^N$  Transition Model
- Analysis and Geometric Sensitivities

## session03/naca.csm

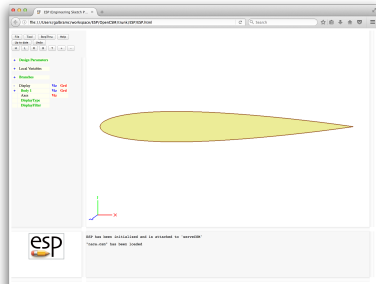
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```
# NACA design paramters
DESPMTR  thick  0.12      #frac of local chord
DESPMTR  camber 0.00      #frac of local chord

# Construct the airfoil
UDPRIM   naca  Thickness thick  Camber camber
ATTRIBUTE capsAIM $xfoilAIM;mSESAIM;tsfoilAIM

# output the area
OUTPMTR  Area
SET      Area @area
```

---



## MSES AIM Documentation

- MSES Overview
- Accessing Functional Derivatives
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## Derivatives or OUTPMTR w.r.t DESPMTR

### session03/1\_mses\_Deriv.py

```
filename = "naca.csm"
print ("\n==> Loading geometry from file \""+filename+"\"...")
capsProblem = pyCAPS.Problem(problemName = "design_mses",
                             capsFile = filename,
                             outLevel = 0)

# Alias the geometry
naca = capsProblem.geometry

print('\n==> Area and Area derivatives...')
print("--> Area      ", naca.outpmtr["Area"].value)
print("--> dArea/dthick ", naca.outpmtr["Area"].deriv("thick"))
```

- Create MSES AIM and set analysis inputs

## session03/1\_mses\_Deriv.py

---

```
# Setup MSES
mses = capsProblem.analysis.create(aim = "msesAIM", name="mses")

# Set flow condition
mses.input.Alpha = 0.0
mses.input.Mach = 0.5
mses.input.Re = 5e6

# Set meshing parameters
mses.input.GridAlpha = 0
mses.input.Airfoil_Points = 201
```

---



- Desired derivative variables set with Design\_Variable input
  - MSES always computes derivatives w.r.t. Alpha, Mach, Re,
- Output derivatives w.r.t. Analysis and Geometric inputs accessed via deriv method

## session03/1\_mses\_Deriv.py

```
# Geometric derivative variables
mses.input.Design_Variable = {"camber":{}, "thick":{}}

# Print the functional and gradients
print('\n==> Drag and Drag derivatives...')
print( "--> CD          =", mses.output["CD"].value )
print( "--> dCD/dAlpha  =", mses.output["CD"].deriv("Alpha") )
print( "--> dCD/dMach   =", mses.output["CD"].deriv("Mach") )
print( "--> dCD/dcamber =", mses.output["CD"].deriv("camber") )
print( "--> dCD/dthick  =", mses.output["CD"].deriv("thick") )

print('\n==> All Drag derivatives...')
print( "--> CD derivs   =", mses.output["CD"].deriv() )
```

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- Create MSES AIM and set analysis inputs

## session03/2\_mses\_DerivPlot\_CD\_Alpha.py

---

```
# Setup MSES
mses = capsProblem.analysis.create(aim = "msesAIM", name="mses")

# Set flow condition
mses.input.Alpha = 0.0
mses.input.Mach = 0.5
mses.input.Re = 5e6

# Set meshing parameters
mses.input.GridAlpha = 0
mses.input.Airfoil_Points = 201

# Trip the flow near the leading edge to get smooth gradient
mses.input.xTransition_Upper = 0.1
mses.input.xTransition_Lower = 0.1
```

---

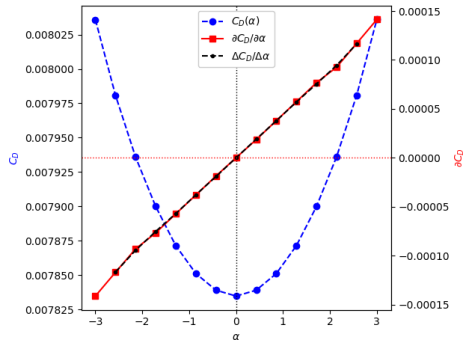
- Analytic vs. Central Difference derivatives
- Plot vs. Alpha
  - $C_d$ ,  $\partial C_d / \partial \alpha$ ,  $\Delta C_d / \Delta \alpha$

## session03/2\_mses\_DerivPlot\_CD\_Alpha.py

```
# Plot the functional and gradient
Alphas = np.linspace(-3, 3, 15)
CD = []
CD_Alpha = []
for Alpha in Alphas:
    print("--> Alpha", Alpha)
    mses.input.Alpha = Alpha

    CD.append(      mses.output["CD"].value )
    CD_Alpha.append(mses.output["CD"].deriv("Alpha" ) )

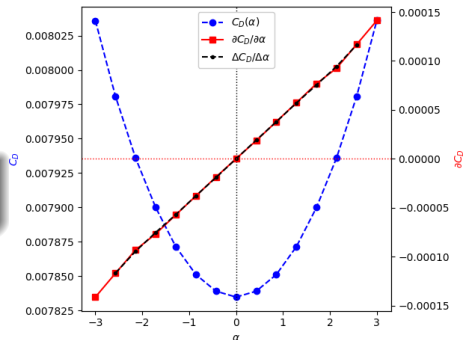
# Compute central difference derivatives
dCD_Alpha = []
for i in range(1,len(Alphas)-1):
    dCD_Alpha.append( \
        (CD[i+1]-CD[i-1])/(Alphas[i+1]-Alphas[i-1]))
```



- Gradient based optimization with OpenMDAO<sup>1</sup>

$$\min_{\alpha} C_D \quad (1)$$

- Symmetric NACA 0012
- Exact solution:  $\alpha = 0$



<sup>1</sup><https://openmdao.org>

## ● Basic analysis inputs

### session03/3\_mses\_OpenMDAO\_CD\_Alpha.py

---

```
capsProblem = pyCAPS.Problem(problemName="design_mses",
                             capsFile="naca.csm",
                             outLevel=0)

# Setup MSES
mses = capsProblem.analysis.create(aim = "msesAIM", name="mses")

# Set flow condition
mses.input.Alpha = 3.0    # Initial guess away from solution Alpha == 0
mses.input.Mach = 0.5
mses.input.Re = 5e6

# Set meshing parameters
mses.input.GridAlpha = 0
mses.input.Airfoil_Points = 201

# Trip the flow near the leading edge to get smooth gradient
mses.input.xTransition_Upper = 0.1
mses.input.xTransition_Lower = 0.1
```

---

- Define output functional and partial derivatives

session03/3\_mses\_OpenMDAO\_CD\_Alpha.py

---

```
class msesAnalysis(om.ExplicitComponent):

    def initialize(self):
        # Setup arguments for the component.

        # CAPS Problem input
        self.options.declare('capsProblem', types=object)

    def setup(self):
        # Assign initial values to the variables.

        capsProblem = self.options['capsProblem']
        mses = capsProblem.analysis['mses']

        # attach parameters to OpenMDAO object
        self.add_input('Alpha', val=mses.input.Alpha)

        # Add output metric
        self.add_output('CD')

        # Declare and attach partials to self
        self.declare_partials('CD', 'Alpha')
```

---

- Set *alpha* based on “inputs” from OpenMDAO
- Return functional  $C_D$  with “outputs”

### session03/3\_mses\_OpenMDAO\_CD\_Alpha.py

```
def compute(self, inputs, outputs):  
    # Compute functionals  
  
    capsProblem = self.options['capsProblem']  
    mses = capsProblem.analysis['mses']  
  
    print("--> Alpha", inputs['Alpha'])  
  
    # Update input values if changed  
    if mses.input.Alpha != inputs['Alpha']:  
        mses.input.Alpha = inputs['Alpha']  
  
    # Grab objective and attach as an output  
    outputs['CD'] = mses.output.CD  
    print("--> CD", outputs['CD'])
```



- Return  $\partial C_D / \partial \alpha$  with “partials”

### session03/3\_mses\_OpenMDAO\_CD\_Alpha.py

```
def compute_partials(self, inputs, partials):
    # Compute functional partial derivatives

    capsProblem = self.options['capsProblem']
    mses = capsProblem.analysis['mses']

    print("--> Alpha", inputs['Alpha'])

    # Update input values if changed
    if mses.input.Alpha != inputs['Alpha']:
        mses.input.Alpha = inputs['Alpha']

    # Get derivatives and set partials
    partials['CD', 'Alpha'] = mses.output["CD"].deriv("Alpha")
    print("--> CD_alpha", partials['CD', 'Alpha'])
```

session03/3\_mses\_OpenMDAO\_CD\_Alpha.py

---

```
# Setup the openmdao problem object
omProblem = om.Problem()

# Create the OpenMDAO component
msesSystem = msesAnalysis(capsProblem = capsProblem)

# add subsystem to model
omProblem.model.add_subsystem('msesSystem', msesSystem)

# add design variables to model
omProblem.model.add_design_var('msesSystem.Alpha', lower=-5, upper=5)

# add objective to minimize CD
omProblem.model.add_objective('msesSystem.CD')

# setup the optimization
omProblem.driver = om.ScipyOptimizeDriver()
omProblem.driver.options['optimizer'] = "L-BFGS-B"
omProblem.driver.options['tol'] = 1.e-9
omProblem.driver.options['disp'] = True

# Press go
print("\n==> Starting Optimization...")
omProblem.setup()
omProblem.run_driver()
omProblem.cleanup()

print("--> Optimized value:", omProblem.get_val("msesSystem.Alpha"))
```

---

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- Analytic vs. Central Difference derivatives
- Plot vs. camber
  - $C_d$ ,  $\partial C_d / \partial \text{camber}$ ,  $\Delta C_d / \Delta \text{camber}$

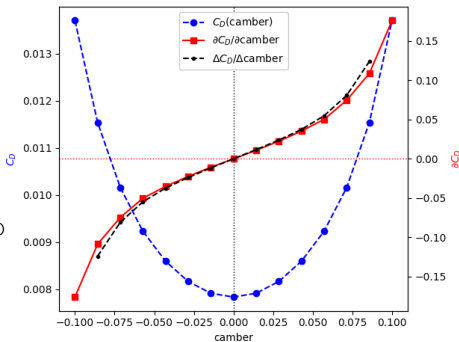
session03/4\_mses\_DerivPlot\_CD\_camber.py

```
# Use camber as the design variable
mses.input.Design_Variable = {"camber":{}}

# Plot the functional and gradient
cambers = np.linspace(-0.1, 0.1, 15)
CD = []
CD_camber = []
for camber in cambers:
    print("--> camber", camber)
    capsProblem.geometry.despmtr.camber = camber

    CD.append(      mses.output["CD"].value )
    CD_camber.append(mses.output["CD"].deriv("camber" ) )

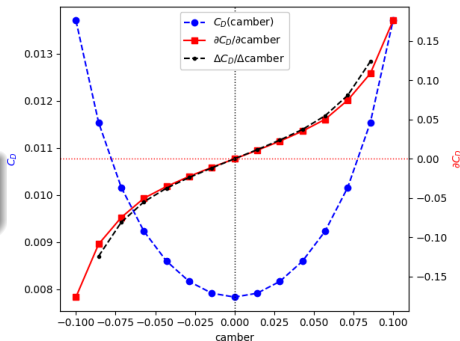
# Compute central difference derivatives
dCD_camber = []
for i in range(1,len(cambers)-1):
    dCD_camber.append( \
        (CD[i+1]-CD[i-1])/(cambers[i+1]-cambers[i-1]))
```



- Gradient based optimization with OpenMDAO

$$\min_{\text{camber}} C_D \quad (2)$$

- NACA Airfoil
- Exact solution: camber = 0



- Basic analysis inputs

## session03/5\_mses\_OpenMDAO\_CD\_camber.py

---

```
# Set a non-zero camber. The optimization should drive it back to zero
capsProblem.geometry.despmtr.camber = 0.1
```

```
# Setup MSES
mses = capsProblem.analysis.create(aim = "msesAIM", name="mses")
```

```
# Set flow condition
mses.input.Alpha = 0.0
mses.input.Mach = 0.5
mses.input.Re = 5e6
```

```
# Set meshing parameters
mses.input.GridAlpha = 0
mses.input.Airfoil_Points = 201
```

```
# Trip the flow near the leading edge to get smooth gradient
mses.input.xTransition_Upper = 0.1
mses.input.xTransition_Lower = 0.1
```

```
# Compute derivatives w.r.t. camber
mses.input.Design_Variable = {"camber":{}}
```

---

- Define output functional and partial derivatives

session03/5\_mses\_OpenMDAO\_CD\_camber.py

---

```
def setup(self):
    # Assign initial values to the variables.

    capsProblem = self.options['capsProblem']
    naca = capsProblem.geometry

    # attach parameters to OpenMDAO object
    self.add_input('camber', val=naca.despmtr.camber)

    # Add output metric
    self.add_output('CD')

    # Declare and attach partials to self
    self.declare_partials('CD', 'camber')
```

---

- Set camber based on “inputs” from OpenMDAO
- Return functional  $C_D$  with “outputs”

session03/5\_mses\_OpenMDAO\_CD\_camber.py

---

```
def compute(self, inputs, outputs):
    # Compute functionals

    capsProblem = self.options['capsProblem']
    naca = capsProblem.geometry
    mses = capsProblem.analysis['mses']

    print("--> camber", inputs['camber'])

    # Update input values if changed
    if naca.despmtr.camber != inputs['camber']:
        naca.despmtr.camber = inputs['camber']

    # Grab objective and attach as an output
    outputs['CD'] = mses.output.CD
```

---



- Return  $\partial C_D / \partial \text{camber}$  with “partials”

session03/5\_mses\_OpenMDAO\_CD\_camber.py

```
def compute_partials(self, inputs, partials):
    # Compute functional partial derivatives

    capsProblem = self.options['capsProblem']
    naca = capsProblem.geometry
    mses = capsProblem.analysis['mses']

    print("--> camber ", inputs['camber'])

    # Update input values if changed
    if naca.despmtr.camber != inputs['camber']:
        naca.despmtr.camber = inputs['camber']

    # Get derivatives and set partials
    partials['CD', 'camber'] = mses.output["CD"].deriv("camber")
    print("--> CD_camber", partials['CD', 'camber'])
```

session03/5\_mses\_OpenMDAO\_CD\_camber.py

---

```
# Create the OpenMDAO component
msesSystem = msesAnalysis(capsProblem = capsProblem)

# add subsystem to model
omProblem.model.add_subsystem('msesSystem', msesSystem)

# add design variables to model
omProblem.model.add_design_var('msesSystem.camber', lower=-0.1, upper=0.1)

# add objective to minimize CD
omProblem.model.add_objective('msesSystem.CD')

# setup the optimization
omProblem.driver = om.ScipyOptimizeDriver()
omProblem.driver.options['optimizer'] = "L-BFGS-B"
omProblem.driver.options['tol'] = 1.e-5
omProblem.driver.options['disp'] = True

# Press go
print("\n==> Starting Optimization...")
omProblem.setup()
omProblem.run_driver()
omProblem.cleanup()

print("--> Optimized value:", omProblem.get_val("msesSystem.camber"))
```

---

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- Gradient-Based Optimization with Geometric Inputs
- **Gradient-based Optimization for Structures**
- Suggested Exercises

- Gradient based optimization with OpenMDAO
- Analysis Input: Wing shell membraneThickness ( $\delta_w$ )

session03/6\_masstran\_OpenMDAO\_thick.py

```
shellWing = {"propertyType"      : "Shell",  
            "membraneThickness" : 0.1,  
            "material"          : "unobtainium"}  
masstran.input.Property = {"wing" : shellWing, "htail": shellTail,  
                           "vtail": shellTail, "fuse" : shellFuse}
```

- Optimization statement

$$\min_{\delta_w} (X_{cg}(\delta_w) - X_{cg\ design})^2 \quad (3)$$

- $X_{cg}(\delta_w)$  computed with masstran
- $X_{cg\ design}$  specified desired CG location (F-118D.csm)

- CAPS uses the NASTRAN model for design variables
- Independent “Design Variable”:  $x_1, x_2, x_3 \dots$
- Dependent “Design Variable Relation”
  - Links independent design variables to NASTRAN Inputs
  - constantCoeff:  $c_0$
  - linearCoeff:  $c_1, c_2 \dots$

$$\delta_w(x_1, x_2, x_3 \dots) = c_0 + \sum_{i=1} c_i x_i \quad (4)$$

session03/6\_masstran\_OpenMDAO\_thick.py

---

```
shellFuse = {"propertyType"      : "Shell",
             "membraneThickness" : 0.2,
             "material"          : "madeupium"}

shellWing = {"propertyType"      : "Shell",
            "membraneThickness" : 0.1,
            "material"          : "unobtainium"}

masstran.input.Property = {"wing" : shellWing, "htail": shellTail,
                          "vtail": shellTail, "fuse" : shellFuse}

# Specify design variables
masstran.input.Design_Variable = {
    "x1": {"initialValue": shellWing["membraneThickness"]},
    "x2": {"initialValue": 0},
}

# Set design variable relations
relationWing = {"componentType": "Property",
               "componentName": "wing",
               "fieldName" : "membraneThickness",
               "constantCoeff" : 0.0,
               "variableName" : ["x1", "x2"],
               "linearCoeff" : [1.0, 0.1]}

relationFuse = {"componentType": "Property",
               "componentName": "fuse",
               "fieldName" : "membraneThickness",
               "constantCoeff" : 0,
               "variableName" : "x1",
               "linearCoeff" : 1.0}

masstran.input.Design_Variable_Relation = {"wingRel": relationWing, "fuseRel": relationFuse}
```

---

session03/6\_masstran\_OpenMDAO\_thick.py

---

```
masstran = capsProblem.analysis.create(aim = "masstranAIM", name="masstran")

# Set mesh generation inputs
masstran.input.Edge_Point_Min = 20
masstran.input.Edge_Point_Max = 20
masstran.input.Quad_Mesh = True

# Set materials
madeupium = {"materialType" : "isotropic",
             "density"      : 10}
unobtainium = {"materialType" : "isotropic",
               "density"      : 25}
masstran.input.Material = {"madeupium": madeupium, "unobtainium": unobtainium}

# Set properties
shellTail = {"propertyType" : "Shell",
             "membraneThickness" : 0.5,
             "material" : "unobtainium"}
shellFuse = {"propertyType" : "Shell",
             "membraneThickness" : 0.2,
             "material" : "madeupium"}
shellWing = {"propertyType" : "Shell",
             "membraneThickness" : 0.1,
             "material" : "unobtainium"}
masstran.input.Property = {"wing" : shellWing, "htail": shellTail,
                           "vtail": shellTail, "fuse" : shellFuse}
```

---

- Define output functional and partial derivatives

## session03/6\_masstran\_OpenMDAO\_thick.py

---

```
def setup(self):
    # Assign initial values to the variables.

    capsProblem = self.options['capsProblem']
    geometry = capsProblem.geometry
    masstran = capsProblem.analysis['masstran']

    # attach parameters to OpenMDAO object
    self.add_input('x1', val=masstran.input.Design_Variable["x1"]["initialValue"])
    self.add_input('x2', val=masstran.input.Design_Variable["x2"]["initialValue"])

    # Add output metric
    self.add_output('dCGx')

    # Declare and attach partials to self
    self.declare_partials('dCGx', 'x1')
    self.declare_partials('dCGx', 'x2')
```

---



- Return functional dCGx with “outputs”

## session03/6\_masstran\_OpenMDAO\_thick.py

```
def compute(self, inputs, outputs):
    # Compute functionals

    capsProblem = self.options['capsProblem']
    geometry = capsProblem.geometry
    masstran = capsProblem.analysis['masstran']

    print("--> x1", inputs['x1'])
    print("--> x2", inputs['x2'])

    # Update input values
    if masstran.input.Design_Variable["x1"]["initialValue"] != inputs['x1'] or \
        masstran.input.Design_Variable["x2"]["initialValue"] != inputs['x2']:
        masstran.input.Design_Variable = {
            "x1": {"initialValue": float(inputs['x1'])},
            "x2": {"initialValue": float(inputs['x2'])},
        }

    x_cg_d = geometry.despmtr.xcg
    x_cg = masstran.output.CG[0]

    # Grab objective and attach as an output
    outputs['dCGx'] = (x_cg - x_cg_d)**2
    print("--> dCGx", outputs['dCGx'])
```

## session03/6\_masstran\_OpenMDAO\_thick.py

```
def compute_partials(self, inputs, partials):
    # Compute functional partial derivatives

    capsProblem = self.options['capsProblem']
    geometry = capsProblem.geometry
    masstran = capsProblem.analysis['masstran']

    print("--> x1", inputs['x1'])
    print("--> x2", inputs['x2'])

    # Update input values
    if masstran.input.Design_Variable["x1"]["initialValue"] != inputs['x1'] or \
        masstran.input.Design_Variable["x2"]["initialValue"] != inputs['x2']:
        masstran.input.Design_Variable = {
            "x1": {"initialValue": float(inputs['x1'])},
            "x2": {"initialValue": float(inputs['x2'])},
        }

    x_cg_d = geometry.despmtr.xcg
    x_cg = masstran.output.CG[0]

    cg_x1 = masstran.output["CG"].deriv("x1")[0]
    cg_x2 = masstran.output["CG"].deriv("x2")[0]

    # Get derivatives and set partials
    partials['dCGx', 'x1'] = 2*(x_cg - x_cg_d)*cg_x1
    partials['dCGx', 'x2'] = 2*(x_cg - x_cg_d)*cg_x2
    print("--> dCGx_x1", partials['dCGx', 'x1'])
    print("--> dCGx_x2", partials['dCGx', 'x2'])
```

## Thickness Derivatives

- Plot  $C_d$  and  $\partial C_d / \partial \text{thick}$ 
  - Start from a copy of `session03/4_mses_DerivPlot_CD_camber.py`

## CG Optimization with Geometry Variables

- Optimize wing:xroot and htail:lh in F-118D such that the CG computed with masstran matches the design CG

## session03/f118-D.csm

# F-118D Boxster

DESPMTR    xcg                      9    # Desired x cg location

# wing design parameters

```

DESPMTR    wing:area                30    # area
DESPMTR    wing:aspect            9.00   # aspect ratio
DESPMTR    wing:thick              0.10   # thickness ratio
DESPMTR    wing:xroot              4.0    # xloc at root LE
DESPMTR    wing:zroot             -0.5   # zloc at root LE

```

# horizontal tail design parameters

```

DESPMTR    htail:lh                5    # htail moment length
DESPMTR    htail:aspect            4.15   # htail aspect ratio
DESPMTR    htail:thick            0.08   # htail thickness
DESPMTR    htail:Vh                0.8    # htail volume coeff
DESPMTR    htail:zroot            0.5    # zloc of root LE

```

# vertical tail design parameters

```

DESPMTR    vtail:aspect            1.80   # vtail aspect ratio
DESPMTR    vtail:tau               0.08   # vtail thickness
DESPMTR    vtail:Vv                0.03   # vtail volume coeff
DESPMTR    vtail:yroot            0    # yloc of root LE

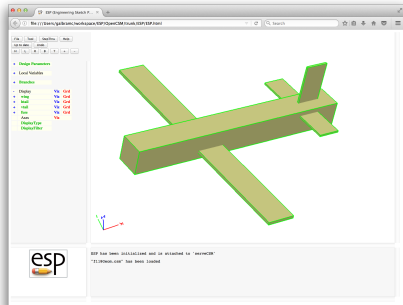
```

# fuselage design parameters

```

DESPMTR    fuse:width              1.2    # vtail area
DESPMTR    fuse:height            1.2    # vtail aspect ratio

```



Move wing and tail s.t. CG  
from masstran matches  
design CG

- Htail and Vtail sized by tail volume coefficient

$$V_h = \frac{S_h L_h}{S_w C_w} \rightarrow S_h = \frac{V_h S_w C_w}{L_h}$$

- $L_h$  and  $X_w$  are free design parameters

session03/f118-D.csm

```
#=====
# Htail
SET      htail:area      htail:Vh*wing:area*wing:chord/htail:lh
SET      htail:span      sqrt(htail:aspect*htail:area)
SET      htail:chord      htail:area/htail:span
SET      htail:xroot      xcg+htail:lh-htail:chord/4

BOX      htail:xroot      -htail:span/2      htail:xroot      htail:chord      htail:span      htail:chord*htail:thick
#=====
# fuselage
SET      fuse:length      xcg+htail:lh+3*htail:chord/4

BOX      0      -fuse:width/2      -fuse:height/2      fuse:length      fuse:width      fuse:height
```

- Gradient based optimization with OpenMDAO
- Geometry Input variables: wing:xroot, htail:lh

$$\min_{\text{wing:xroot,htail:lh}} (X_{cg} - X_{cg\,design})^2 \quad (4)$$

- $X_{cg}$  computed with masstran
- $X_{cg\,design}$  specified desired CG location (F-118D.csm)